



## Uncertainty calculation in transport models and forecasts.

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# **Uncertainty calculation in transport models and forecasts**

PhD Thesis

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## **Abstract**

Transport projects and policy evaluations are often based on transport model output, i.e. traffic flows and derived effects. However, literature has shown that there is often a considerable difference between forecasted and observed traffic flows. This difference causes misallocation of (public) resources, hence resulting in socio-economic losses. Along with technical and decision-process related issues, such inaccuracy also originates from transport models' inherent uncertainty, which in turns originates from the complexity of the systems generating both transport supply (e.g. services, infrastructure, and regulation) and demand. Uncertainty pertains to everything the modeller does not know to a full extent about the system object of the modelling process due to a limited knowledge or stochasticity of some model components. Thus, ultimately uncertainty reflects the inability of the modeller to represent the complex system in a deterministic way.

By modelling complex systems, transport models are subject to uncertainty. The main consequence of such uncertainty is that point estimates of modelled traffic flows, and their derived measures, only represent one of the possible outputs generated by the model. Analyses based on point estimates invariably produce uncertain results and decisions taken relying on them may easily lead to unexpected consequences. Thus, it is essential to assess uncertainty inherent to transport models. This requires producing uncertainty measures by investigating which are the main sources of uncertainty within the model, how uncertainty propagates throughout the model and, finally, how it affects the model output.

The purpose of the studies described in this thesis was to investigate uncertainty inherent to transport models. Despite its importance, the relation between the uncertainty of the transport model components and that of transport models output, and the processes that govern such relation, are not often explored by the existing literature. The collection of the four papers that compose the present thesis fills some of the gaps of this study area. The analyses were implemented by using an approach based on stochastic techniques (Monte Carlo simulation and Bootstrap re-sampling) or scenario analysis combined with model sensitivity tests. Two transport models are used as case studies: the Næstved model and the Danish National Transport Model.

The first paper investigated the effects of uncertainty in the volume-delay function parameters used in the Danish National Transport Model<sup>1</sup>. The results showed that some links in the modelled network have high sensitivity to the variability in the function parameters. In particular, the affected links mainly refer to short, mid-distance road types potentially hosting commuting traffic. Any assessment of projects potentially affecting traffic flow on those links should then take into consideration this sensitivity and integrate uncertainty analysis in the decision process.

The second paper analysed the uncertainty in a four-stage transport model related to different variable distributions (to be used in a Monte Carlo simulation procedure), assignment procedures and levels of congestion, at both the link and the network level. The analysis used as case study the Næstved model, referring to the Danish town of Næstved<sup>2</sup>. The results highlighted that both the choice of the variable distributions and the use of different assignment algorithms has a noticeable impact on model output. Besides, it showed that the higher the link congestion, the lower the level of final uncertainty.

The third paper presented in this thesis deals with uncertainty in transport demand forecasts. In particular, the uncertainty in the socio-economic variables (population, GDP, employment and petrol prices) growth rate projections is investigated and a method is suggested to assess its propagation throughout time. The analysis used as case study the Danish National Transport Model<sup>3</sup>. The resulting model output uncertainty was neither linear nor similar for the different model outputs investigated. Transport related projects may focus on different model outputs which have a different temporal uncertainty propagation patterns. Thus, making acknowledgeable the uncertainty propagation pattern over time specific for key model outputs becomes strategically important.

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<sup>1</sup> Manzo, S., Nielsen, O. A. & Prato, C. G. (2014). The Effects of uncertainty in speed-flow curve parameters on a large-scale model. *Transportation Research Record*, 1, 30-37.

<sup>2</sup> Manzo, S., Nielsen, O. A. & Prato, C. G. (2015). How uncertainty in input and parameters influences transport model output: a four-stage model case-study. *Transport Policy*, 38, 64-72.

<sup>3</sup> Manzo, S., Nielsen, O. A. & Prato, C. G. (2015). How uncertainty in socio-economic variables affects large-scale transport model forecasts. *Forthcoming: European Journal of Transport and Infrastructure Research*, 15-3, 64-72.

The last paper<sup>4</sup> examined uncertainty in the spatial composition of residence and workplace locations in the Danish National Transport Model. Despite the evidence that spatial structure influences travel behaviour, there is no consensus on the strength of such influence. To provide insights to this topic, the study investigated a number of possible future scenarios affecting the spatial structure. Among the others, the observed trend of increasing population in the major Danish cities and the variation of employment location scenarios were analysed. The results show that the combined effect of higher urban density and social mobility produces an increase in number of trips; of these, density seems to be the dominant factor. However, at the same time, the proximity of the destinations increases, so decreasing the average trip length and consequently the total mileage travelled.

Overall, results from the studies collected in this thesis visibly show the importance of integrating in a systematic way uncertainty analysis in transport modelling frameworks. This should be a standard approach to produce the information necessary to increase the quality of the decision process and to develop robust or adaptive plans. In fact, project evaluation processes that do not take into account model uncertainty produce not fully informative and potentially misleading results so increasing the risk inherent to the decision to be taken. Uncertainty analysis, by allowing identifying the main sources of uncertainty within the model and by providing knowledge on the level of confidence of the model output, ultimately enhances the robustness of the travel demand models and of the analyses based on their output.

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<sup>4</sup> Working paper.

## Abstract in Danish

Transportprojekter og politiske vurderinger er ofte baseret på input fra trafikmodeller, dvs. trafikstrømme og afledte effekter. Det fremgår imidlertid af litteraturen, at der ofte er betydelig forskel mellem de forventede og observerede trafikstrømme. Denne forskel medfører uhensigtsmæssig anvendelse af (offentlige) midler og har dermed også samfundsøkonomiske konsekvenser. Udover tekniske og beslutningsrelaterede forhold skyldes denne unøjagtighed også den i trafikmodellerne iboende usikkerhed. Hver gang der bygges en model med henblik på at reproducere et komplekst system, vil outputtet herfra uvægerligt blive påvirket af usikkerhed. Usikkerhed vedrører alt det, som modeludvikleren ikke har fuldt kendskab til i det system, der er genstand for modelleringsprocessen, på grund af begrænset viden om visse af modellens komponenter eller deres stokasticitet. Usikkerheden afspejler således i sidste ende modeludviklerens manglende evne til at gengive det komplekse system på en deterministisk måde.

Når man modellerer komplekse systemer, er trafikmodellerne underlagt usikkerhed. Den vigtigste konsekvens af denne usikkerhed er, at punktestimaterne i de modellerede trafikstrømme og deres afledte foranstaltninger kun er et af modellens flere mulige outputs. Analyser, der er baseret på punktestimater, vil uvægerligt give usikre resultater, og beslutninger, der træffes på denne baggrund, vil ofte få uventede konsekvenser. Det er derfor vigtigt at vurdere usikkerheden ved trafikmodeller og etablere foranstaltninger, der tager højde for usikkerheden, ved at undersøge, hvad der er hovedårsagerne til modellens usikkerhed, hvordan usikkerheden spreder sig gennem modellen, og endelig hvordan den påvirker modellens output.

Formålet med de undersøgelser, der er beskrevet i denne afhandling, var at undersøge usikkerheden ved trafikmodeller. Til trods for dets betydning er forholdet mellem usikkerheden i tilknytning til trafikmodelkomponenterne og trafikmodeloutputtet og de processer, der styrer dette forhold, ikke beskrevet særligt indgående i den eksisterende litteratur. De fire artikler, der udgør denne afhandling, udfylder nogle emner inden for dette område. Undersøgelserne blev gennemført ved hjælp af en tilgang baseret på stokastiske teknikker (Monte Carlo-simulering og Bootstrap re-sampling) og scenarieanalyse i kombination med følsomhedstests af modellen. Der er anvendt to trafikmodeller som case study: Næstved-modellen og Landstrafikmodellen.

Det første studie undersøgte effekterne af usikkerheden i forbindelse med parametrene i de speed-flowkurver, der anvendes i Landstrafikmodellen.<sup>5</sup> Resultaterne viste, at visse af kanterne på vejnettet i det modellerede netværk har en høj følsomhed over for ændringer i parametrene. Det er især kanterne på korte, mellemdistance vejtyper, der potentielt anvendes til pendlertrafik, der bliver berørt. Enhver vurdering af projekter, der kan berøre trafikstrømmen på disse kanter, bør derfor tage hensyn til denne følsomhed og lade usikkerhedsanalysen indgå i beslutningsprocessen.

Det andet studie undersøgte usikkerheden i en firtrins-trafikmodel ved forskellig fordeling af variabler (skal anvendes i en Monte Carlo-simuleringsprocedure), rutevalgsmodeller og trængselsniveauer på såvel netværks- som kantniveau. Analysen brugte Næstvedmodellen, der omfatter Næstved med opland, som case study<sup>6</sup>. Resultaterne understregede, at såvel valget af fordeling af variabler som anvendelsen af forskellige assignment-algoritmer har mærkbar indvirkning på modellens output. Det viste endvidere, at jo højere trængslen på kantniveau er, desto lavere er det endelige usikkerhedsniveau.

Den tredje artikel i denne afhandling drejer sig om usikkerheden i forbindelse med transportefterspørgselsprognoser. Det er hovedsageligt usikkerheden i de samfundsøkonomiske variabler (befolkning, BNP, beskæftigelse og benzinpriser), der bliver undersøgt, og der foreslås en metode til at vurdere spredningen af denne usikkerhed over tid. Undersøgelsen brugte Landstrafikmodellen som case study<sup>7</sup>. Usikkerheden ved modellens output var hverken lineær eller ensartet for de forskellige undersøgte outputs fra modellen. Transportrelaterede projekter kan fokusere på forskellige modeloutputs, som har forskellige mønstre for usikkerhedsspredning over tid. Det er derfor strategisk vigtigt at anskueliggøre det specifikke usikkerhedsspredningsmønster over tid for de vigtigste modeloutputs.

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<sup>5</sup> Manzo, S., Nielsen, O. A. & Prato, C. G. (2014). The Effects of uncertainty in speed-flow curve parameters on a large-scale model. *Transportation Research Record*, 1, 30-37.

<sup>6</sup> Manzo, S., Nielsen, O. A. & Prato, C. G. (2015). How uncertainty in input and parameters influences transport model output: a four-stage model case-study. *Transport Policy*, 38, 64-72.

<sup>7</sup>Manzo, S., Nielsen, O. A. & Prato, C. G. (2015). How uncertainty in socio-economic variables affects large-scale transport model forecasts. *Forthcoming: European Journal of Transport and Infrastructure Research*, 15-3, 64-72.

Den sidste artikel<sup>8</sup> undersøgte usikkerheden i forbindelse med den rumlige sammenknytning af bopæle og arbejdspladser i Landstrafikmodellen. Selvom det er påvist, at rumlig struktur påvirker transportadfærden, er der ikke enighed om omfanget heraf. For at få nærmere viden om dette emne undersøgte en række mulige fremtidsscenarier, der kan påvirke den rumlige struktur. Den observerede tendens med en voksende befolkning i de større danske byer og ændringen i arbejdspladsplaceringer blev undersøgt. Resultaterne viser, at den kombinerede effekt af højere bytæthed og øget social mobilitet øger antallet af ture, og at bytæthed ser ud til at være den mest dominerende faktor. Det ser dog samtidig ud til, at afstanden til destinationerne bliver kortere, hvorved den gennemsnitlige turlængde falder og hermed det samlede kørselsomfang.

Overordnet set fremgår det tydeligt af de resultater, der er indsamlet i studierne i denne afhandling, at det er vigtigt systematisk at medtage usikkerhedsanalyse i forbindelse med trafikmodellering. Det bør være en standardmetode til at tilvejebringe den information, der er nødvendigt for at øge kvaliteten af beslutningsprocessen og udvikle robuste eller adaptive planer. Projektvurderingsprocesser, der ikke tager højde for modelusikkerhed, giver således ufyldstgørende og potentielt misvisende resultater, hvilket øger risikoen ved den beslutning, der skal træffes. Usikkerhedsanalyser øger således robustheden af efterspørgselsmodellerne og de analyser, der baserer sig herpå, ved at identificere de vigtigste årsager til usikkerhed i modellen og ved at give viden om modeloutputtets konfidensniveau.

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<sup>8</sup> Working paper.



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## 1 Introduction

The present thesis is about uncertainty in transport models and forecasts. Transport models consist of equations combining exogenous variables and coefficients that express how the endogenous variable, the model output, depends on exogenous variables (De Jong et al., 2007). Transport models play a prominent role in many decision-making processes. The different outputs they generate, ultimately traffic flows and derived measures, are the key input for a wide range of policy analyses, such as appraisals for new infrastructures, urban development planning strategy and sustainable mobility policy. However, transport models have been often criticized for the lack of accuracy in their output and an extensive literature has shown that there is often a considerable difference between forecasted and observed traffic flows (e.g., Bain, 2003; Bain and Plantagie, 2004; Bain and Polakovic, 2005; Flyvbjerg, 2005; Flyvbjerg et al., 2006; Parthasarathi and Levison, 2010; Welte and Odeck, 2011).

This inaccuracy has been often ascribed to technical and decision-process related issues. However, the list of potential sources of transport models inaccuracy should include the complexity of the modelled transport systems, as suggested by Van Zuylen et al. (1999)<sup>9</sup>. Complex systems are highly structured and not straightforward to describe in their dynamics, thus unpredictable in their output. Whenever a model is created to reproduce a complex system, its output will invariably be affected by uncertainty. Uncertainty pertains to everything the modeller does not know to a full extent (Van Geenhuizen et al., 1998) due to limited knowledge or stochasticity of some model components and is then inherent to the modelling process.

By modelling complex systems, transport models are subject to uncertainty. Besides, the components generating and affecting transport systems come from the economic environment as much as from social, psychological and technological environments. All these are complex systems with inherent uncertainty, which transport systems absorb through the incoming inputs. Thus, transport models are affected by uncertainty in all their components. When considering, for instance, the definition of the model context boundaries, the data collection process, and the parameter calibration of the stochastic variables, they all contain a certain degree of uncertainty which is reflected in the model output.

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<sup>9</sup> More in details, the list suggested by Van Zuylen includes both complexity in the internal dynamics of the system and in the dynamics of the interactions between the system and its external environment.



Inherent uncertainty prevents from modelling with a deterministic approach in the sense that it is unrealistic to describe transport systems as functioning through deterministic processes. This dramatically reduces the reliability of transport models point output, irrespective of the technical quality and un-biasedness of the model. Transport models output expressed as point estimates simply do not contain enough information to be safely used as input for transport project and policy assessments. Furthermore, their use not only limits the reliability of the analyses implemented, but it is also conceptually incorrect. In fact, although a point estimate output can result to be accurate, it is only one of the possible outputs originated by a model. Indeed, the use of range and confidence estimates rather than point estimates is suggested for describing knowledge about uncertain situations (Rowe, 1994). In fact, as pointed out by Kahneman and Tversky “The prevalent tendency to underweight, or ignore, distributional information is perhaps the major source of error of intuitive prediction (...) The analyst should therefore make every effort to frame the forecasting problem so as to facilitate utilizing all the distributional information that is available to the expert.” (Kahneman and Tversky, 1977). With this respect, uncertainty analysis allows not only defining the sources of uncertainty within a model and how they affect the model output, but also describing uncertainty margins of the model output.

The present thesis investigates uncertainty in transport models by addressing the issue from different perspectives. However, all the analyses were run using a similar approach based on stochastic techniques or scenario analysis combined with model sensitivity tests. The present thesis includes four papers:

- The first paper investigates the effects of uncertainty in the parameters of the volume-delay function used in the assignment model of the Danish National Transport Model (NTM).
- The second paper examines the uncertainty in a four-stage transport framework, modelling the traffic of the Danish town of Næstved. The focus is on the role of variable distribution, assignment algorithms and levels of congestion on the final model output uncertainty.
- The third paper is about uncertainty in transport model forecasts. In particular, the paper deals with uncertainty in the forecasts of socio-economic variables which are used as inputs by the NTM to model future traffic scenarios. Not only the future values of the socio-economic variables but also the demographic trends have inherent uncertainty.

- The last paper deals with the effects of uncertainty in the future changes of the spatial distribution of the residence and workplaces locations in Denmark on the demand of transport. Different hypotheses are made and tested by using the NTM.

The thesis is organized as follows. Chapter 2 provides the theoretical background for the overall work, by defining the sources and the taxonomy of uncertainty in modelling and then the specific issues related to transport modelling uncertainty. At the end, the chapter illustrates the methods commonly applied in transport modelling uncertainty analyses. Chapter 3 describes in details the two transport models used as case study for this research: the Næstved model and the NTM. The chapters from 4 to 7 present the four papers that compose the present thesis. Finally, Chapter 8 summarizes the main conclusions.

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## 2 Uncertainty in transport modelling; theory and methods

### 2.1 The sources of uncertainty in modelling

A model can be synthetically defined as an abstraction of a system of interest<sup>10</sup>. Systems consist of set of elements, the system components, which interact through a structure of connections eventually producing an output called emerging behaviour. Systems are separated from their surrounding environment by boundaries. However, systems called adaptive are able to interact with the surrounding environment through incoming inputs which then become system components themselves. The aim of the modelling is to reproduce the structure connecting the system components and, hence, the system output.

As summarized in Table 2-1, systems can be divided between simplistic and complex. Simplistic systems are characterized by deterministic dynamics which make the emerging behaviour predictable. Typically, simplistic systems can mostly be found in the fields of physics and chemistry. In contrast, complex systems are those systems whose components interact in a way that is difficult to understand, thus making the emerging behaviour difficult to predict. More precisely, their (true) emerging behaviour cannot be computed with precision due to lack of knowledge. In fact, complex systems are characterised by stochastic processes. A process is defined stochastic (from the Greek *στόχος*, for aim or guess) when the process (subsequent) state is determined both by the process predictable development and by random elements.

**Table 2-1. Systems classification.**

Systems		
Category	Simplistic	Complex
Process type	Deterministic	Stochastic
Emerging behaviour	Predictable	Unpredictable

As a consequence of complexity, whenever a model is created to reproduce a complex system will invariably be affected by unpredictability, which reflects the inability of the modeller to represent the complex system with a deterministic approach. Unpredictability refers to

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<sup>10</sup> This is true for all the possible kind of models: verbal, physical, graphical or mathematical. This thesis focuses on mathematical models: "The purpose of mathematical modelling is to depict (in mathematical form, ed.) causal interdependencies in a real or imagined reality in such a way that certain insights of importance for the planning task at hand are obtained." (Leleur, 2000)

ignorance, uncertainty, or their combined effect, and ultimately reflects the lack of knowledge of the modeller about the emerging behaviour of the modelled system. Ignorance refers to a complete absence of awareness of some model components or their dynamics, thus remaining confined outside the modelling process. As previously pointed out, uncertainty instead pertains to everything the modeller does not know about the system to a full extent.

## **2.2 The taxonomy of uncertainty**

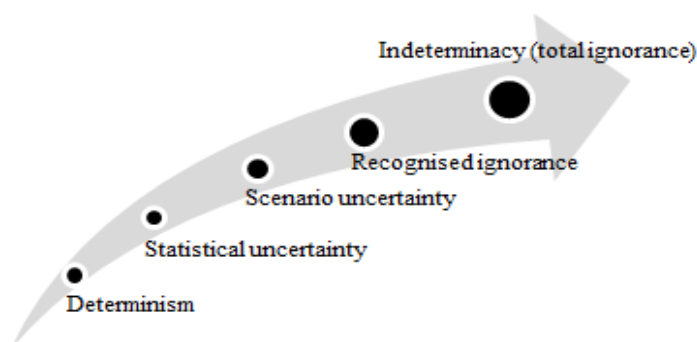
The existing literature usually defines uncertainty through classification. The resulting picture tends to be somehow unclear, given that the taxonomy and terminology proposed vary depending on the author and the scientific discipline. This thesis mainly uses the taxonomy of uncertainty proposed by Walker et al. (2003). Walker suggests a general definition of uncertainty as being “any departure from the unachievable ideal of complete deterministic knowledge of the relevant system”. Furthermore, it classifies uncertainty by using three dimensions: nature, level and location.

The *nature of uncertainty* can be epistemic or ontological (variability). Epistemic uncertainty originates from the imperfection of the modeller’s knowledge of the context or the functioning of the model, and can theoretically be reduced by more research and empirical work<sup>11</sup>. Ontological uncertainty, instead, is due to the stochasticity or randomness associated with some model components. Unlike epistemic uncertainty, ontological uncertainty is inherent to the model components and cannot be reduced; nevertheless, it is normally possible to analyse it to achieve a better understanding of its dynamics. Epistemic and ontological uncertainties usually coexist, thus, a complete model uncertainty analysis should address both. It is worth to stress that dealing with uncertainty in modelling means trying to obtain a better understanding of its dynamics and awareness of its consequences rather than trying to eliminate it. In fact, pure deterministic condition cannot be achieved in the context of complex systems modelling. It can be probably said that the right way to cope with uncertainty is to use information from uncertainty analyses to develop adaptive or robust plans.

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<sup>11</sup> However, epistemic uncertainty does not necessarily decrease with acquired knowledge. In fact, an increase of knowledge of the system, e.g. due to new available information, can increase the level of uncertainty by revealing new lack of knowledge or new sources of uncertainty previously unknown (van Asselt and Rotmans, 2002).

The *levels of uncertainty* are shown in Figure 2-1 which represents three increasing levels – statistical uncertainty, scenario uncertainty, and recognized ignorance – bounded between determinism and indeterminacy. Statistical uncertainty is defined as any uncertainty that can be adequately described in statistical terms, that is when it is assumed possible to define all the (continuous) outputs of an uncertain event along with their probability of occurrence (Refsgaard et al., 2007). Scenario uncertainty refers instead to uncertain events whose output are discrete and have no related probability of occurrence. Typically, it refers to the effects of changes in the external context on the system, especially with regard to future scenarios. Finally, recognized ignorance reflects the modeller’s awareness of a lack of knowledge preventing from representing uncertainty at both statistical and scenario level. With respect to the bounds, both determinism and total ignorance lie outside the range of uncertainty. While determinism excludes uncertainty by definition, the reason why total ignorance is excluded from the range of uncertainty as well is that uncertainty is not related to what is totally unknown, but refers to something that is not entirely understood<sup>12</sup>.



**Figure 2-1. Levels of uncertainty (modified after Walker et al. 2003).**

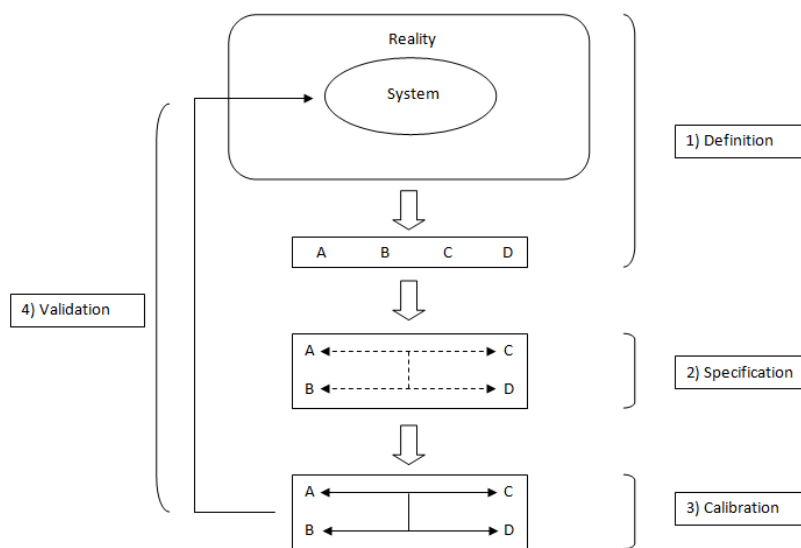
With respect to the *location of uncertainty* within the four steps of a generic modelling process (i.e. model definition, specification, calibration and validation), such as the one graphically described in Figure 2-2, all three levels of uncertainty described above might affect the following locations:

<sup>12</sup> Refsgaard et al. (2007) included a level called “qualitative” between scenario uncertainty and recognized ignorance to define any uncertainty which is not numerically quantifiable but whose existence and effects can be included in the analysis through, for instance, expert elicitation. Instead, non-quantifiable uncertainty, ignorance and indeterminacy should be left entirely to the decision-making process (Harraemoes and Madsen, 1999).

- Model definition
  - Context: uncertainty in defining the system boundaries to separate the system from the surrounding environment (in Figure 2-2, identifying the area within the reality space where the system operates) and in identifying the main drivers of the processes operating the system (in Figure 2-2, identifying components from A to D).
- Model specification
  - Structure: uncertainty about the assumed model structure (in Figure 2-2, represented as dash lines connecting components from A to D).
- Model calibration
  - Inputs (data collected): uncertainty in quantifying the present and, when required, future values of the model drivers identified in the model definition step (in Figure 2-2, quantifying components from A to D identified as inputs).
  - Parameters (calibrated variables): uncertainty in calibration methods applied to quantify parameters (i.e. computing and methodological issues) and in choosing assumed or imported parameters (in Figure 2-2, quantifying components from A to D identified as parameters).
- Model validation
  - Output: uncertainty propagated throughout the model and finally manifested in the model output. The model output does not have an inherent uncertainty itself but absorbs the uncertainty propagated by other locations.
  - Data: uncertainty in the collection processes implemented to collect the data used for the validation. Indeed, the less reliable the data collected are the less reliable is the model validation process and then its capacity to testing the accuracy of the model and its acceptability for the intended use.

With respect to the model context and structure uncertainty, there are two specific problems. Firstly, underlying all (probability) models is the belief that the future will behave as the past (Rowe, 1994). In other words, there is an implicit assumption of persistence through time of the model context and structure, whilst instead future scenarios would often require changing both context and structure to adapt the model to the new conditions (Hodges, 1987). Secondly, only predictable and recurrent phenomena can be included in a model, but unpredictable (thus non-recurrent) events, might have a stronger impact on the modelled output, especially when the

model is used for forecasting purposes. While unpredictable events cannot be included in the model by definition, non-recurrent events would define the model up to a potentially undesirable over specification. Besides, including more variables for a better fit increases the model complexity and the uncertainty of the model output (Walker et al., 2003)<sup>13</sup>.



**Figure 2-2. Modelling process diagram (adapted from Leleur 2000).**

### 2.3 Uncertainty in transport models

Both epistemic and ontological uncertainty affects transport models at statistical, scenario and recognised ignorance level. Statistical uncertainty mainly refers to the model inputs and parameters. As previously pointed out, this is the level of uncertainty that can be better analysed given the possibility to describe it through probabilistic analysis. Scenario uncertainty instead concerns events, mainly affecting the context where the modelled transport system operates, for which no probability of occurrence can be defined. Examples can be taken from the socio-economic environment (e.g. trends of spatial distribution) or the political environment (e.g. transport policies). These events may or may not happen and different scenarios can be hypothesized. However, it is not possible to define a related probability of

<sup>13</sup> Besides, it also decreases the flexibility of the model itself. At the contrary, the key characteristics of the model should be flexibility and parsimony (Harraemoes and Madsen, 1999).



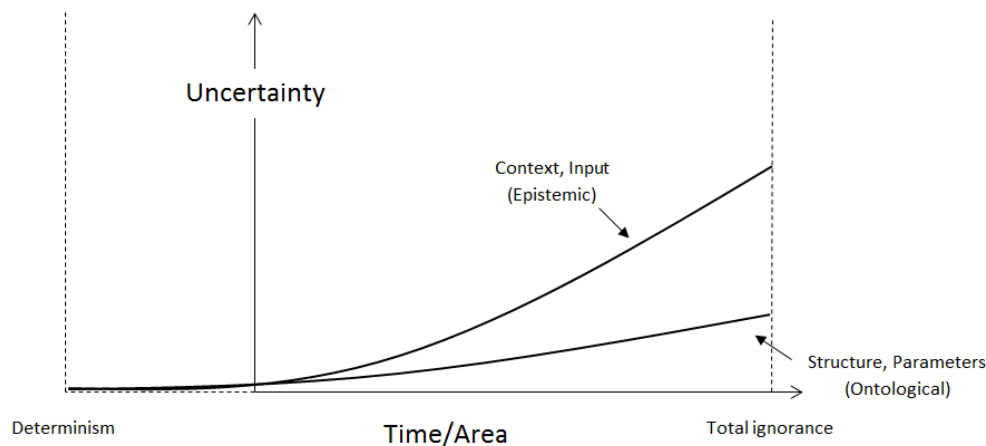
occurrence, so that a probabilistic analysis cannot be implemented. Finally, recognized ignorance can be related to major and unpredictable changes in the system environment, such as unexpected technological breakthroughs.

With respect to the location, uncertainty in transport models can be summarized as follows:

- Model definition
  - Context: uncertainty related to the definition of the boundaries of the system being modelled, such as the number of alternative modes to include or the geographical size of the area of interest. In particular, when modelling long-term demand, a change in the context would often be required due to future changes in the transport demand environment, such as new technologies or real estate development.
- Model specification
  - Structure: uncertainty about the assumed model structure. Moreover, the technical implementation of the model presents some inherent uncertainty related to the methodology chosen, such as the number of iterations to run.
- Model calibration
  - Input (data collected): uncertainty related to the data collection processes and to the future values of the inputs. The exogenous variables, such as population and income are themselves output of complex systems and thus unpredictable in their future values.
  - Parameters (calibrated variables): uncertainty related to the estimation of model parameters. It mainly refers to the use of estimated parameters from samples rather than from population. Moreover, transport models might include assumed or imported parameters, with inherent uncertainty. Parameters uncertainty also refers to the specification errors in the model equations.
- Model validation
  - Output: uncertainty related to the transport model output. This refers to the process by which uncertainty located in various parts of the model is transferred to the model output. In particular, transport models can be structured as frameworks including several models estimated and applied sequentially; each sub-model output is used as input for the subsequent model. Each step includes uncertainty which propagates throughout the overall model.

- Data: uncertainty in the collection processes implemented to collect the data used for the validation.

With regard to the influence on the final output, it can be argued that the prevalence of some uncertainty location on the others depends on the model time dimension, such as graphically described in Figure 2-3. The influence of model structure and parameter uncertainty is higher in static or short run forecast models whilst context and input uncertainty influence increases in long run models, as results from Matas et al. (2012) suggest. In fact, modelling observed processes guarantees a fairly precise knowledge of the model context and inputs values, whose uncertainty, in this time dimension, is mainly epistemic, thus reducible, for instance through better data collection. As a consequence, model structure and parameter uncertainty have a higher impact on the model output. In the long run forecasts instead the uncertainty related to the model context and input tends to prevail. The further in the future the model forecasts the more different the context can be from the one observed and reproduced in the model. The same problem affects the model input values, such as socio economic variables. In other words, even if ontological uncertainty is assumed constant throughout time, the epistemic component of uncertainty, for instance related to the system context, increases due to the future changes in the overall system.



**Figure 2-3. Time and geographical dimension relation with uncertainty.**

The same considerations debated with respect to the time dimension can be applied to the geographical size. Modelling systems which refer to vast geographical areas implies dealing with a high number of inputs, and related data collection issues. Similarly, the model context

becomes increasingly unknown and undefined, subject to a long chain of events and decisions. By modelling at the national or regional level, not only national but also international socio-economic changes, political decisions and trade flows may become relevant in shaping the model context. As a consequence, input and context uncertainty have a relative higher effect on model output uncertainty. Vice versa, modelling small areas guarantees reliable data collection and accurate context knowledge, thus increasing the relative impact of the model structure and parameters uncertainty on model output uncertainty<sup>14</sup>.

The combined effect on model output uncertainty of the modelled time dimension and geographical size produces higher uncertainty in modelling demand of transport for major infrastructures. In fact, the financial feasibility of major infrastructures relies on long-term traffic forecasts and their context has usually to be defined at national or international level, so increasing the overall model output uncertainty.

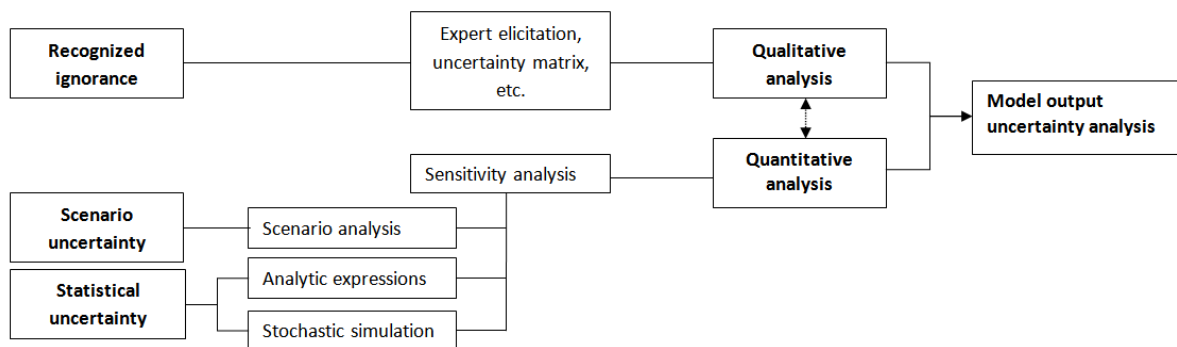
## **2.4 Uncertainty in transport models: methods of analysis**

Many studies investigating uncertainty in transport models use an approach based on model sensitivity analysis, consistently with what suggested, among others, by Ashley (1980). Sensitivity analysis quantifies the impact on the model output of a variation occurred in one or more model components. This allows understanding the relation between the modified component and the model output, in terms, for instance, of relevance and linearity. In transport modelling the number of methodologies applied to investigate uncertainty is small as compared to other fields of research. For instance, with respect to environmental modelling processes, Refsgaard et al. (2007) reviewed 14 different methods. Instead, according to the results from the literature reviews conducted by De Jong et al. (2007) and Rasouli and Timmermans (2012), the number of methodologies commonly used for transport modelling uncertainty analyses is lower. With respect to quantitative approaches, the number of methodologies commonly used

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<sup>14</sup> However, it is worth to notice that what wrote about the time and geographical dimension relation with uncertainty might of course have exceptions. For instance, some socio-economic inputs might be easier to predict in the long run, due to the time smoothing process. Similarly, some geographical areas might be affected by, for instance, political decisions and trade flows relatively more than the nation where they are located as a whole.

can be broadly summarized in the following three<sup>15</sup>: scenario analysis, analytic expressions and stochastic simulation. Referring to the levels of uncertainty, whilst scenario analysis is implemented at scenario uncertainty level, both analytic expressions and stochastic simulation methods can be applied in presence of uncertainty at statistical level. For this reason, these techniques are mainly used to assess uncertainty in the model inputs and parameters (hereinafter mainly referred to as “variables”) which, following Walker’s taxonomy, have inherent uncertainty at statistical level. The three methods use different techniques to quantify uncertainty in the model variables but then they all use sensitivity analysis to quantify the effects of such uncertainty in the model output. The qualitative approaches, such as expert elicitations or uncertainty matrix (Walker et al. 2003) provide a method to deal with uncertainty in presence of recognized ignorance. Both approaches aim to identify the major sources of uncertainty in the model, although without quantify them. Qualitative and quantitative methods, graphically summarized in Figure 2-4, can be seen as complementary. The qualitative methods can in fact provide information to narrow down the analyses to be implemented through the quantitative methods and even, with a margin of approximation, validate them.



**Figure 2-4. Common methodologies used to implement uncertainty analyses.**

All the analyses implemented in this thesis use quantitative methods. The order in which the three different methodologies are presented in the next three paragraph corresponds to the (increasing) level of information they are able to provide to support the decision making

<sup>15</sup> The three methods described do not complete the range of (theoretically) possible methods available. Among the others, for instance, it is worth to remember the work of Sevcikova et al. (2007) using a Bayesian melding approach.

processes. The analyses presented in this thesis were implemented by using sensitivity scenario analysis and stochastic simulation. Both the Næstved model and the NTM are in fact not suitable for the analytic expression approach<sup>16</sup>.

### **Scenario analysis**

In the scenario analysis approach, the model sensitivity tests are implemented based on modeller's hypotheses about the uncertainty of the model components. These hypotheses may refer either to best guess estimates assigned to uncertain numerical variables or to characteristics of other model components, such as infrastructural changes in the network. Scenario analysis requires a two steps process. Firstly, the modeller has to make the hypotheses with respect to the value of the model component(s) under investigation, for example, in case of forecasting, about their future value. Usually more than one scenario is produced, including two extremes and a most likely case for each of the variables investigated. These hypotheses may either be based on expert elicitation – from the modeller or other experts or stakeholders – or from the analysis of available data. Secondly, sensitivity tests are run in order to produce different model output for the different hypothesized scenarios.

Scenario analysis has two main downsides. Firstly, it usually implies a subjective judgment with respect not only to which model components to analyse but also to the specification of the scenario to test. Secondly, and more critically, scenario analysis does not provide probability of occurrence of the variation hypothesised and of the corresponding model output. These two characteristics make scenario analysis somehow close to a deterministic approach, in the sense that it produces a number of model point output without probability of occurrence. On the other hand, it is probably the only reasonable approach for analyses at scenario uncertainty level, for example those focusing on future changes of the network design or on the implementation of alternative transport policies. A few studies applied a methodology related to scenario analysis, such as Rodier and Jhonston (2002) in their work referring to socio-economic variables uncertainty in model forecasts.

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<sup>16</sup> As described more in details in the following of this thesis, the analytic expression approach requires to be implemented on models with unique solution. Neither the Næstved model nor the NTM have such characteristic.

## **Analytic expressions**

The analytic approach is based on the use of analytic expressions to derive the variance of the endogenous variables, i.e. the model output, resulting from the variance of the exogenous variables, i.e. inputs and parameters estimates. More in details, the analytic method combines the information about the derivatives of the model output to input and parameters with the variance-covariance matrix of inputs/parameters. Using this information is then possible to estimate the variance-covariance matrix of the model output and then their confidence intervals and their correlation with inputs and parameters (Yang et al. 2013).

The analytic expression method can only be implemented on models with a unique solution. Thus, the uncertainty inherent to frameworks combining different models, such as the classical four-stage model, cannot be addressed with this approach. More in general, this method offers advantages if the model equations are relatively straightforward (De Jong et al. 2007), otherwise it loses part of its effectiveness. In particular, in presence of non-straightforward model equations, the analytic expression method might become a cumbersome process, then losing its main advantage as compared to stochastic simulation, which is the quick implementation time.

Apart from the limitations discussed, the main downside of the analytical method is that it can only provide variance and coefficient of variation (and derived measures) whilst stochastic simulation methods allow generating the distribution of the entire model output. In other words, the amount of information about the uncertainty of the model output that the analytic expression method is able to provide is limited when compared to that provided by stochastic simulation<sup>17</sup>. The analytic expression method has been used regularly in transport model uncertainty literature, such as in Leurent (1998) and Yang et al. (2013). A theoretical discussion is also presented by Ben-Akiva and Lerman (1985).

## **Stochastic simulation**

In stochastic simulation the value of the model variables under investigation, to be used in the sensitivity tests, is the result of a stochastic sampling process. In fact, through stochastic

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<sup>17</sup> One possible way to overcome this issue is to combine analytic expressions approach with stochastic simulation by (i) calculating the variance of the exogenous variables analytically and then (ii) assuming normal distribution and drawing from it.

sampling, the model uncertainty deriving from the variables uncertainty is quantified by substituting frequency distributions for the deterministic variable values (Alcamo and Bartnicki, 1987). Unlike model context, structure and technical implementation, model inputs and parameter uncertainty can be described through a probability distribution function and so they are suitable for stochastic simulation. Stochastic simulation allows not only a less invasive judgment from the modeller, because of the randomness of the stochastic sampling process which generates the draws, but also the possibility to match each (sampled) variable value with a probability of occurrence. This allows defining a likelihood of occurrence for the corresponding model output generated through the sensitivity tests. Therefore, also the model output results configured as a probability distribution. In fact, through stochastic sampling a potentially infinite number of possible values of the variables under investigation can be estimated and used for the sensitivity tests. The result of this procedure, unlike the analytic expression method, is the generation of a probability distribution of the model output, which is of key importance in providing an informative support for decision processes. This is because unlike the scenario analysis, which gives equal weight to all the simulated scenarios, the probability distribution resulting from the stochastic sampling defines the margins of likelihood for the output ranges so allowing differentiating the model output in the high or low probability regions. Broadly speaking, it is possible to divide stochastic simulation techniques applied in transport modelling studies in two categories: Monte Carlo Simulation (MCS) and re-sampling techniques<sup>18</sup>.

MCS has been extensively used in transport modelling uncertainty analysis, such as in Ashley (1980), Zhao and Kockelman (2001), Pradhan and Kockelman (2002), Krishnamurty and Kockelman (2003), De Jong et al. (2007), and Zhang et al. (2011). The MCS consists in two steps. First, stochastic sampling is implemented to obtain a number of random draws from the probability distribution of the variables under investigation. Afterwards, model sensitivity tests are run using the drawn values to obtain the corresponding probability distribution of the model output. To run the stochastic sampling, three are the information required: the central value, the dispersion and the probability distribution of the variables examined. Both central value and dispersion can usually be calculated through the analysis of available data, such as

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<sup>18</sup> However, MCS and re-sampling techniques can be combined by using re-sampling to define the variable distribution and then draw from it with MCS.

time series. The main issue with the MCS technique is the choice of the variables probability distribution to be used in the stochastic sampling procedure. In fact, if in one hand this is critically important given its effects on the final output (Armoguum, 2003) on the other hand it is far from being an easy task. The distributions mostly applied in transport modelling uncertainty literature to implement MCS are the normal and lognormal distributions:

- The normal distribution is a family of bell shaped asymptotic distributions which differ in the value of their two parameters:  $\mu$ , defining the location (mean value) of the distribution, and  $\sigma$ , defining the scale (standard deviation) of the distribution. In case of  $\mu = 0$  and  $\sigma = 1$  the distribution is called standard normal distribution. The normal distribution main characteristics are that it is symmetric around the mean and that mean, mode and median coincide. The normal distribution can be very useful in probabilistic simulations. Many model variables can be assumed normally distributed, such as socio-economic variables growth rates, or are normally distributed by definition, such as errors in some parameter calibration methods. However, when normal distribution is used to implement stochastic sampling, the asymptotic shape of its tails has to be taken into account. The risk is in fact to draw values which are unrealistic, because either too high or too low, or meaningless, because, for instance, negative when the (sampled) variable cannot take a negative value.
- The lognormal distribution is a family of continuous distributions of variables whose logarithm is normally distributed. The lognormal distribution is defined by the same two parameters of the normal distribution,  $\mu$  and  $\sigma$ . Unlike the normal distribution though, the lognormal distribution's domain ranges from 0 to  $+\infty$  and is positively skewed. This characteristic makes lognormal distribution particularly suitable for sampling variables which need a (positive) sign constraint. Another characteristic of the lognormal distribution is that when  $\sigma$  is small compared to  $\mu$ , the skew is small and the distribution approaches a normal distribution. As a consequence, any normal distribution can be approximated by a lognormal by using the same standard deviation but increasing the mean, so that the ratio  $\sigma/\mu$  is small.

Despite the normal and lognormal distributions are by far the most common distributions used in transport modelling uncertainty studies, different distributions can prove to be useful to investigate uncertainty in transport models variables. Among the others, the following distributions deserve attention and have also been, although sporadically, applied:



- Rectangular. The key characteristic of the rectangular distribution, also called uniform distribution, is that the probability of occurrence is the same for all the values that the variable can take within the chosen domain, defined by a minimum and maximum value. For this reason, rectangular distribution does not provide a very good insight of a variable uncertainty. However, this downside can be reversed and used as a point of strength whenever there is no available data or knowledge to infer a more suitable distribution.
- Triangular. The triangular distribution is defined by three parameters corresponding to the mode and the minimum and maximum values of the distribution. Although the mode can be defined out of expert elicitation or available data, defining the boundaries can be more difficult. Likewise the rectangular distribution, the triangular distribution is typically used when there is a lack of information about the population but, unlike the rectangular distribution, there is an expectation on the population most likely value (the mode).
- Gamma. The Gamma distribution is a family of continuous probability distributions right skewed and bound to zero. The Gamma distribution is a two parameters distribution, where the parameters are both positive real numbers. There are three different possible parameterizations: (i) shape parameter  $k$  and a scale parameter  $\theta$ , (ii) shape parameter  $\alpha = k$  and an inverse scale parameter  $\beta = 1/\theta$ , called a rate parameter, and (iii) shape parameter  $k$  and a mean parameter  $\mu = k/\beta$ . The Gamma distribution can be very useful because of its flexibility given that it can take many different shapes, from exponential (when  $k=1$ ) to normal (for increasing values of  $k$ ).

Besides the choice of the variables probability distribution, another issue related to the MCS approach raises from the assumptions, or the lack of assumptions, regarding the correlation between the variables to be used in the sampling procedure, whenever the analysis involves more than one variable. This problem, however, can usually be overcome: for model input, correlation analysis can be implemented using data time series, while calibration procedures provide model parameters correlation matrixes. Whenever none of these options are available, then modeller's judgment is required to define variables correlation. In this case, however, this has to be clearly stated to make the decision makers aware of the additional uncertainty deriving from such approach.

The MCS requires a large number of samples, up to 10000, in order to well represent the distribution of the variables under investigation. This is because the draws are randomly taken

along the distribution and so only a high number of them can guarantee a good representation of all the areas of the distribution. If, on the one hand, this preserves the purity of the randomness which is the rationale of the stochastic sampling approach, on the other hand it implies a high number of model sensitivity tests, which result in time consuming processes. To increase the computational efficiency of the sampling procedure, different sampling methods have been introduced, such as Latin Hypercube Sampling (LHS) and factorized design approach. LHS stratifies the variables probability distribution by dividing the cumulative curve into equal intervals and then takes one random value from each interval; the effect is that there is no longer pure random sampling but instead stratified random sampling. As a consequence, it is possible to represent the distribution very closely with a lower number of draws. Factorized design is based on the same rationale of LHS in the sense that the variables probability distribution is divided into equal intervals. However, unlike LHS, it selects the mid-percentile of each interval as value for the sampling process<sup>19</sup>.

Re-sampling techniques can be used to assess uncertainty in calibrated parameters, whenever the sample and the model used for the calibration are available. Two are the main random re-sampling techniques applied in transport modelling uncertainty literature: Jack's knife (Quenouille, 1949) and Bootstrap (Efron and Tibshirani, 1993). Both approaches allow overcoming the necessity of the modeller judgment in defining the parameter distributions by creating an "observed" parameter distribution through repeated model runs on model subsamples. In other words, they infer the uncertainty of a parameter  $\beta$  based on the observed variability of  $\beta$  resulting from several calibrations implemented on the model sub-samples. Both methods have been applied in transport modelling to investigate on parameter uncertainty.

Jack's knife approach evaluates the uncertainty of  $\beta$  by re-calibrating  $\beta$  using a number of sub-samples equal to  $n + 1$ , where  $n$  is the sample size. The sub-samples are created by subtracting from the original sample one (or more) observation at a time. From this new set of replicates of  $\beta$ , it is possible to quantify the bias and the variance of the parameter. As pointed out by Hugusson (2005), although Jack's knife only require  $n + 1$  computations, thus resulting computationally efficient, it results reliable only when the statistic  $\beta$  is approximately linear.

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<sup>19</sup> Along with LHS and factorized design approach, another sampling method used in transport uncertainty analysis literature is Halton draws, implemented for instance in De Jong et al. (2007).

Jack's knife has been used to define confidence intervals of variables, such as in Armoogum (2003).

Bootstrap method, the most commonly used among the two, investigates the accuracy of  $\beta$  starting from the initial assumption of considering the original sample, originating  $\beta$ , as the population. Bootstrap consists in a three steps procedure. First, from the original sample of  $n$  observations a number of samples, also called Bootstrap samples, are generated through (re)sampling with replacement. All samples contain  $n$  observations as the original one. The replacement approach guarantees that each observation in the original sample has a constant probability  $1/n$  to be drawn and placed in the new generated samples so that these differ from each other. Second,  $\beta$  is calculated for each Bootstrap sample. Finally, the new  $\beta$  values obtained are analysed to infer the accuracy of the estimator by using uncertainty measures (e.g. variance, standard deviation, confidence intervals or percentiles). Once obtained the  $\beta$  vector, sensitivity tests can be run on the model output. Although straightforward to implement, is important to notice two downsides of the Bootstrap method. First, there is no rule defining the correct number of Bootstrap samples to generate, although the number should be large and in theory tendentially infinite; thus, as for MCS, the sensitivity tests may result in time consuming processes. Second, one limit of Bootstrap technique is that the results are constrained by the quality of the original sample, given that the Bootstrap samples do not increase the amount of information there contained. Bootstrap has been used in many transport uncertainty studies, such as in Brundell-Freij (2000), Hugusson (2005), Matas et al. (2012) and Petrik et al. (2012).

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### 3 Models description

The analyses described in this thesis have been implemented using two models: the Næstved model and the NTM. The Næstved model is a classic four-stage model framework. It has been used to implement the analysis described in chapter 5 dealing with the effects of different sources of uncertainty in a four-stage transport demand model framework. In fact the Næstved model, despite being accurate and including state of art modelling solutions, is a flexible tool that allows to test different hypotheses and to run sensitivity tests in a reasonable amount of time. The NTM has instead a more peculiar structure which makes it closer to an activity based transport model. The NTM has been used for the analyses described in chapters 4,6 and 7 which better fit a large-scale national model. This is due to the NTM characteristics of wide and composite infrastructure (required for the link based speed-flow curve parameters uncertainty analysis), the in depth role of the socio-economic variables (required for the model forecasts uncertainty analysis) and the presence of an integrated land use model (required for the uncertainty in spatial composition analysis).

#### 3.1 The Næstved model

The Næstved model is a four-stage transport model<sup>20</sup> which covers the area of the Danish town of Næstved located in the southern part of Zealand. The four-stage model, graphically shown in the Figure 3-1, is a framework that combines four transport models: trip generation, trip distribution, mode choice and trip assignment<sup>21</sup>. Each model output is used as input for the model that follows; the output from trip assignment is finally used as feedback for a number of iterations which involve trip distribution and the mode choice and, depending on the model, trip generation.

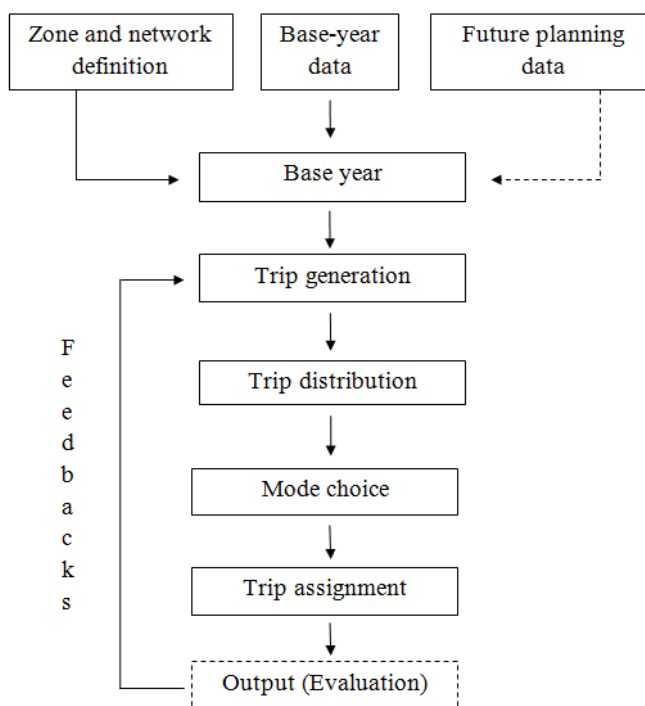
The town of Næstved has a population of around 42,000 increasing to around 80,000 when considering the entire municipality, which has a total surface of around 681km<sup>2</sup>. The total number of trips over a 24h time interval is estimated of around 88,500, 10% of which made by

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<sup>20</sup> The Næstved model is a model built for demonstrative purposes for the transport modelling software Traffic Analyst, licensed by Rapidis ApS.

<sup>21</sup> Transportation modelling and forecasts studies have traditionally followed the sequential four-stage framework, first implemented in the 1950s at the Detroit Metropolitan Area Traffic Study and Chicago Area Transportation Study (CATS). For a more detailed discussion refer, among the others, to Ortuzar and Willumsen, 2011.

public transport through a network of buses connecting Næstved to its urban area, as well as all major surrounding towns. The traffic, modelled over a single 24 hour time interval, is divided in two modes, private and public transport, and in two categories, home/work and business. Due to the small size of the town of Næstved, the network is characterized by low levels of congestion. The final model output is based on iterations which only involve trip distribution, mode choice and trip assignment stages; the trip generation output is kept constant and is not influenced by the travel impedance of the network.



**Figure 3-1. Four-stage transport model.**

### **Baseline data and zone system**

To implement a four-stage model, first the study area, subject to modelling and analysis, is defined. Interactions with the area external to its boundaries – mainly related to: trips to, from and passing through the study area - is modelled via access and egress points. The area of interest is then divided in zones, which need to be homogeneous from land use point of view and population composition; their number varies according to the model purpose and the data availability. In the Næstved model, the area of interest is divided in 106 zones. Afterwards, the



transportation network, which corresponds to the supply side, is represented, typically as a directed graph combining nodes (i.e. junctions) and links (i.e. homogeneous stretches of road between junctions). Both links and nodes have associated attributes (for example, length, speed, and capacity for links and turn prohibitions and penalties for nodes). The network description also includes centroid connectors, which are the abstract links connecting the zones centroids to realistic access points on the physical network. In the Næstved model, the network is composed by 315 links classified as “highway”, “urban” and “local”, which represent respectively around 3%, 5% and 92% of the total number of links. The Figure 3-2 below graphically shows the Næstved model network.

Next, the base-year data related to the study area need to be collected. They usually include:

- Household/person travel surveys: they provide socio-economic and activity/travel data at household/person level and they are mainly used in the model generation, distribution and mode choice stages.
- Traffic studies: they provide information about the network, e.g. traffic counts and link capacity, and they are mainly used in the mode choice and assignment stages and for the overall model validation.



**Figure 3-2. The Næstved model transport network.**

Based on this initial information, the four transport models are sequentially implemented, going from the most aggregated level of demand of traffic (traffic generated by zone) to the most disaggregated (link flows by mode). Trip generation quantifies the propensity to travel in terms of number of trips produced and attracted by each zone. In the trip distribution step, which follows, each trip is allocated to a particular destination (zone), based on the initial travel impedance, i.e. generalized travel cost, of the network. By combining the results of the first two steps the Origin-Destination (OD) trip matrix for the area of interest is produced. The third step, mode choice, involves splitting the overall number of trips to represent proportions of trips by alternative modes. Finally, the trips by mode are assigned to the network in the assignment step. The results of traffic assignment update the travel impedance of the network, modelled as OD cost matrix by mode, which is used to renew calculations of the trip distribution and the mode choice and, depending on the model, trip generation. The iterations end when the network flows reach equilibrium, usually defined as the state when no user can improve his travel condition by unilaterally changing route (user equilibrium).

### Trip generation

In the trip generation the frequency of the trip origins and destinations for each zone is calculated as a function of demographic, land use and other socio-economic factors. The results of this procedure fill the marginal totals in the OD matrix, which represent the total number of trips produced (origin) and attracted (destination) by each zone. In the Table 3-1, modified after Ortuzar and Willumsen (2011),  $O_i$  is the total number of trips produced by zone  $i$ ,  $D_j$  is the total number of trips attracted by zone  $j$  and  $T$  is the total number of trips between all origins  $i$  and destinations  $j$ .

**Table 3-1. Trip generation OD matrix.**

		Destinations							$\sum_j T_{ij}$
		1	2	3	...	j	...	z	
Origins	1								$O_1$
	2								$O_2$
	3								$O_3$
	...								...
	i								$O_i$
	...								...
	Z								$O_z$
	$\sum_i T_{ij}$	$D_1$	$D_2$	$D_3$	...	$D_j$	...	$D_z$	$\sum_{ij} T_{ij} = T$

There are various techniques commonly applied for implementing the trip generation stage, such as cross classification, discrete choice models. The *Næstved model* uses the regression analysis approach, for each unique combination of zone,  $i$ , and category (home/work and business). For trip production, the independent variables are the number of workplaces and workers, WP and W, while trip attraction is based on the number of primary and secondary work places, WPP and WPS, as follows:

$$O_i = \beta_{wp} WP_i + \beta_w W_i + \varepsilon_i \quad (1)$$

$$D_j = \beta_{wpp} WPP_j + \beta_{wps} WPS_j + \varepsilon_j \quad (2)$$

To balance trip generated and attracted a balancing tool is then applied. In fact, irrespectively from which of the three methods is applied, some procedure is usually required to balance the total number of OD trips, given that trips produced and attracted are calculated separately<sup>22</sup>.

### **Trip distribution**

In the trip distribution stage each trip is allocated to a particular destination (zone). In this way, it is possible to fill the internal part of the OD matrix with all the possible  $T_{ij}$ , representing the number of trips between zones  $i$  and  $j$ . In presence of prior knowledge on trip distribution, for instance an already existing trip distribution structure from a previous study, matrix estimation methods can be applied, such as the Furness method. Matrix estimation methods use the existing structure, called initial solution,  $t_{ij}$  and the marginal totals  $O_i$  and  $D_j$  from the trip generation model to calculate the expansion (balancing) factors. The balancing factors are then iteratively applied to  $t_{ij}$ , until the totals by column and rows correspond to the marginal totals, which are the constraints of the procedure.

The main problem with this approach is that it heavily relies on the initial solution, which cannot be modified to simulate, for instance, changes in trip distribution due to changes in the network. Alternatively, a commonly used approach is the gravity model, which produces an initial solution based on the travel impedance of the network. Gravity models guarantee a higher flexibility. In fact, by modulating the initial solution travel impedance, it allows to simulate the effects of changes in the network, such as the variation of travel costs due to pricing policies or infrastructural enhancements. For any  $T_{ij}$  travel impedance is represented by  $c_{ij}$ , the generalized cost of travelling from zone  $i$  to zone  $j$ . In this way the resulting deterrence function  $f(c_{ij})$ , which is inversely proportional to zone separation, provides values for each cell in the OD matrix which substitute  $t_{ij}$  to provide the initial solution. Given  $O_i$ ,  $D_j$  and  $f(c_{ij})$ , the Furness method can be applied and by setting  $t_{ij} = f(c_{ij})$ . Whilst  $a_i$  and  $b_j$  are calibrated throughout the estimation of the model,  $f(c_{ij})$  needs instead to be calibrated independently, usually with power, exponential or combined (Gamma) functions. In the Næstved model the trip distribution stage is based on a gravity model which uses a combined function. The

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<sup>22</sup> Is normal practice to consider trip production models more reliable than trip attraction ones. Therefore,  $D_j$ 's are usually corrected based on  $O_i$ 's.

deterrence value for traffic from zone  $i$  to zone  $j$  for each category (home/work and business) are calculated as:

$$f(c_{ij}) = c_{ij}^{-\eta} \exp(-\theta c_{ij}) \quad (3)$$

The Furness method is then applied to create the OD matrix. As a result of the trip distribution step, the full OD matrix for the area of interest is produced. The OD matrix contains information about the traffic from each zone to all the other zones in the network, as shown in the Table 3-2.

**Table 3-2. Trip distribution OD matrix.**

		Destinations							$\sum_j T_{ij}$
		1	2	3	...	j	...	z	
Origins	1	$T_{11}$	$T_{12}$	$T_{13}$	...	$T_{1j}$	...	$T_{1z}$	$O_1$
	2	$T_{21}$	$T_{22}$	$T_{23}$	...	$T_{2j}$	...	$T_{2z}$	$O_2$
	3	$T_{31}$	$T_{32}$	$T_{33}$	...	$T_{3j}$	...	$T_{3z}$	$O_3$
	...	...	...	...	...	...	...	...	...
	i	$T_{i1}$	$T_{i2}$	$T_{i3}$	...	$T_{ij}$	...	$T_{iz}$	$O_i$
	...	...	...	...	...	...	...	...	...
	z	$T_{z1}$	$T_{z2}$	$T_{z3}$	...	$T_{zj}$	...	$T_{zz}$	$O_z$
	$\sum_i T_{ij}$	$D_1$	$D_2$	$D_3$	...	$D_j$	...	$D_z$	$\sum_{ij} T_{ij} = T$

### Mode choice

The third step of the four-stage framework deals with splitting the  $T_{ij}$  by different modes. Discrete choice models are commonly applied for this purpose<sup>23</sup>. The rationale underlying this approach is that given a mode choice set, the probability that one individual will chose mode  $k$  to travel from  $i$  to  $j$  is a function of the individual's socio-economic characteristics and of the mode  $k$  attractiveness– in terms of travel time, cost, etc. Each individual will try to maximize the utility deriving from choosing one mode as compared to the utility deriving from choosing any other mode(s). The utility of choosing mode  $k$  to travel between  $i$  and  $j$ ,  $U_{ijk}$ , is:

<sup>23</sup> For a more detailed discussion refer, among the others, to Ben-Akiva and Lerman (1985).

$$U_{ijk} = V_{ijk} + \varepsilon_{ijk} \quad (4)$$

where  $V_{ijk}$  represents the measureable part of the utility and it is function of a vector of cost attributes  $c_{ijk}$  specific to the mode and the corresponding parameter  $\delta_c$ , as follows:

$$V_{ijk} = \delta_c c_{ijk} \quad (5)$$

Multinomial Logit (MNL) and Nested Logit (NL) models are commonly used to implement mode choice analyses. The MNL implies perfect substitution among the alternative modes<sup>24</sup> or, in other words, perfect competition. The probability  $P_{ijk}$  that the mode  $k$  is chosen among the  $l$  modes is equal to:

$$P_{ijk} = \frac{\text{Exp}(V_{ijk})}{\sum_l \text{Exp}(V_{ijl})} \quad (6)$$

The NL, instead, assumes some degree of correlation between different modes, which are aggregated in different nests. The competition is then among the nests and among the modes within the (chosen) nest. In the Næstved model the mode choice model is based on an aggregate MNL. The alternative modes are private (car) and public transport (bus).

### **Traffic assignment**

Trips by mode are assigned to the network by the assignment model, which quantifies the traffic flows  $x$  for each link  $r$  part of the network. Given the network conditions (short term period) traffic flows need to be in equilibrium. Equilibrium may be defined by Waldrop's two principles: user equilibrium and system optimum. User equilibrium is reached when no user can improve his travel conditions by unilaterally changing route. System optimum refers instead to a first best condition for the overall network where the average or total travel time within the network is minimized. This implies some network coordination management, for instance through traffic policies such as road pricing.

The underlying assumption of assignment procedures is that the network users behave rationally, so choosing among the available options the route that minimizes the travel time

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<sup>24</sup> This follows from the assumption that  $\varepsilon_{ijk}$  are independent, and identically distributed or, in other words, uncorrelated and homoschedastic.

(and cost)  $TT$  between  $i$  and  $j$ . Given link  $r$  and path  $p$ , it is possible to formulate the user equilibrium problem as a minimization mathematical program (Sheffy, 1984):

$$MinZ = \sum_r \int_{0, x_r} TT_r(x) dx \quad (7)$$

subject to:

$$\sum_p f_{pij} = T_{ij} \quad (8)$$

$$x_r = \sum_p f_{pij} \delta_{rpj} \quad (9)$$

$$f_{pij} \geq 0 \forall p, i, j \quad (10)$$

$$x_r \geq 0 \forall r \quad (11)$$

where  $f_{pij}$  is the flow on path  $p$ , and  $\delta_{rpj}$  is 1 if link  $r$ , on path  $p$ , is used. In practice, drivers might choose different paths while travelling between the same two zones. This behaviour can be ascribed to three main reasons (Ortuzar and Willumsen, 2011): (i) differences in individual perceptions of which is the best route, (ii) different level of knowledge of the possible alternative routes and (iii) congestion effects reflecting the increasing cost of travelling on the best, more attractive, routes. Stochastic methods of traffic assignment are applied to address the issue related to the different drivers' perceptions; the cost of each route is not assumed as perfectly known and univocally perceived but distributed around a mean value.

Three groups of assignment methods are commonly used:

- All or Nothing (AON): deterministic approach, no network capacity constraints.
- User Equilibrium (UE): deterministic approach, network capacity constraints.
- Stochastic User Equilibrium (SUE): stochastic approach, network capacity constraints.

In the Næstved model, trip assignment is based on stochastic user equilibrium<sup>25</sup>: an iterative calculation approach which includes a stochastic simulation of route choice parameters and travel costs and the modelling of capacity constraint. Capacity constraint effects (congestion) are included in the assignment procedure by recalculating the cost of the routes according to

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<sup>25</sup> More precisely, the Næstved model allows choosing between different route assignment methods; the SUE is the method chosen to conduct the analyses described in this thesis.

the different level of traffic assigned after each algorithm's iteration ("modelling capacity restraint"). The Method of Successive Averages (MSA) is an algorithm commonly used to solve both UE and SUE<sup>26</sup>.

The MSA algorithm uses information about travel costs to define individual route choice. Travel costs include monetary cost (e.g. maintenance, tickets, pricing) and travel time cost, which is affected by level of congestion in the network. A commonly used formula to calculate travel time, subject to capacity restraints, is the US Bureau of Public Roads (BPR) (Bureau of Public Roads, 1964):

$$TT_r = FFT_r * \left\{ 1 + \alpha * \left[ \frac{Flow_r + \gamma Flow'_r}{Capacity_r} \right]^\beta \right\} \quad (12)$$

where  $TT_r$  is the total travel time on link  $r$ ,  $FFT_r$  is the free flow time on link  $r$ ,  $Flow_r$  and  $Capacity_r$  refer to the traffic volume and the capacity of link  $r$ ,  $Flow'_r$  refers to the traffic volume on the opposite direction of link  $r$  (relevant only in case of no separated lanes),  $\alpha$  and  $\beta$  are the traffic/delay parameters, and  $\gamma$  represents the effect on speed reduction due to opposite traffic in non-separated lane roads. This formula defines the speed-flow curve, modelling how the travel speed on a road changes as the amount of traffics rises towards the capacity of the road. The travel speed for each link in the model network is recalculated once per iteration, based on the updated model traffic flows.

The results of the traffic assignment algorithm update the travel resistance of the network defining new  $c_{ijk}$  and the chosen route is the one that minimizes the general costs. This information is then used in a feedback process to renew calculations of trip distribution and mode choice. In the *Næstved model* the travel costs to be used in the route choice are calculated as:

$$C_{ijk} = \omega_l L_{ijk} + \omega_{fft} FFT_{ijk} + \omega_{tc} TC_{ijk} + \omega_c c_{ijk} + \varepsilon_{ijk} \quad (13)$$

where the variables are:

$C_{ijk}$  - The generalized cost of travelling by mode  $k$  using link  $r$ ;

$L_{ijk}$  - Length of the link  $r$  by mode  $k$ ;

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<sup>26</sup> AON does not require equilibrium algorithm, given that it converges after a single iteration.



$\omega_l$  – The weight (cost) associated pr. length unit of the route;

$FFT_{ijk_r}$  – The free-flow travel time, incurred when traveling at the allowed speed, without any slow-down caused by capacity-restraints;

$\omega_{ft}$  – The weight (cost) associated pr. unit of time, for free-flow travel time;

$TC_{ijk_r}$  – The congested time, meaning the extra travel time added by congestion to the free-flow travel-time;

$\omega_c$  – The weight (cost) associated pr. unit of time, for additional travel time caused by congestion;

$c_{ijk_r}$  – A monetary cost, if applicable. This might be relevant if, for instance, a toll-road or road pricing scheme is being modelled;

$\omega_c$  – The weight (cost) associated pr. unit of monetary cost (intuitively this should be 1.0);

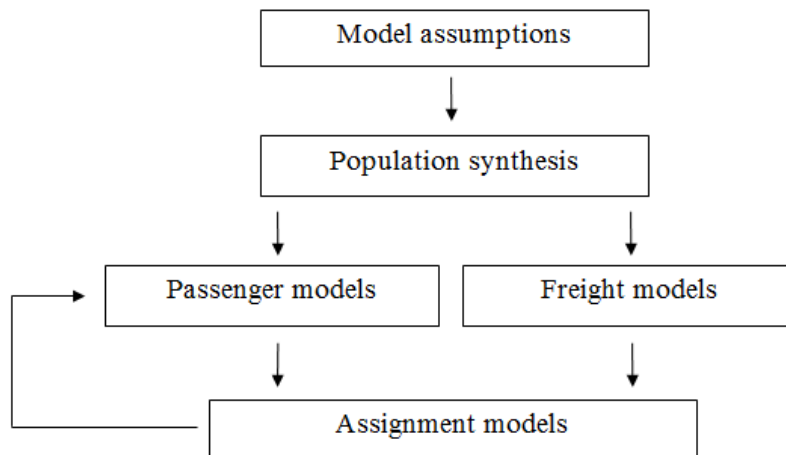
$\varepsilon_{ijk_r}$  is the vector of residuals.

Where stochastic assignment is chosen, the route choice parameters are described as stochastic variables. That is, each route choice parameter is defined as a probability distribution described using distribution function, mean and variance.

### **3.2 The Danish National Transport Model**

The NTM is meant to establish a unified reference model for transport policy analyses and project evaluations in Denmark (Rich et al., 2010). There are several advantages for the development of national transport models. From a decision making process point of view, the use of a reference model allows overcoming the model bias issue rising when different projects are compared based on the output of different models. From a modelling point of view, having a unique reference model implies the convergence of more resources in its development and updating. This permits the development of models more comprehensive and advanced and makes easier to maintain and update the data foundation. The NTM has some features peculiar to activity based models, as it will be further discussed in the following paragraphs. Figure 3-3 below graphically describes the NTM framework. At the initial stage, the model assumptions exogenous to the model, such as demographic, household, employment levels and transport networks, are defined. Then, in the step called population synthesis in the Figure 3-3, a

population matrix is created through the methodology described later. Afterwards, the model consists of two parallel segments: the passenger demand model and the freight demand model. These models feed the assignment models which define the route choice equilibrium; this provides in turn feedbacks to the demand models.



**Figure 3-3. The NTM framework (Adapted from Rich et al. 2010).**

### **Baseline data and zone system**

In the NTM, the zone system is based on four different aggregation levels going from the more aggregated to the more disaggregated: level 0 (municipality level, 98 zones), level 1 (strategic level, 176 zones), level 2 (national level, 907 zones ) and level 3 (regional level, 3670 zones). When constructing the zone system at level 2 and 3 the following constraints were taken into consideration:

- zones should be homogeneous with respect to number of addresses, population and work places;
- proximity to stations should be an identification criterion;
- zones should be connected unambiguously to the road network;
- cities should be distinguished from rural areas (down to 3000 addresses in level 2 and 1000 in level 3);
- special traffic terminals (airports, harbours, transport centres) are defined as individual zones according to their importance;
- zones should also further describe areas with homogeneous land use (industry, apartments, urban centres) – especially at level 3;

- prior zones systems and administrative borders should be taken into consideration.

The NTM is based on two main data sources: the Danish travel survey, namely Transportvane Undersøgelsen (TU), and the Danish national register. TU is a national survey, on-going from 1992, which contains travel information collected monthly from 1000 individuals (increased to 2000 from 2009) which are a representative sample of the Danish population. The survey provides details about daily activity and travel patterns. The national register contains socio-economic information for the entire Danish population, such as age, employment status, income, education, work place and number and type of vehicles (Rich et al., 2010). As described later, this information allows constructing the Danish population matrix according to socio-economic characteristics at both individual and household level and differentiating by residential zones, as described later. Thus, TU and Danish register data combined provide the information to model short term and long term travel decision. In fact the travel information contained in the TU data need to be scaled to the entire population, who's socio-economic characteristics are detailed in the national register.

### **The population synthesis**

The population synthesis model includes in itself two stages. The first is the population fitting stage where the population is fitted according to constraints. The output of the population fitting stage is a “master table” for the population created based on three main target tables (defined exogenously to the model system) for the population, employment and general economic indicators. The second is a simulation stage where the individuals are grouped into households; in fact, the demand models operate on a list of individuals linked into households in order to be able to model household decisions such as car ownership. As a result, the two main tasks of the population synthesis step are (i) to predict the number of individuals within a detailed socio-group conditional on population targets, and (ii) to consistently link these individuals in household entities<sup>27</sup>.

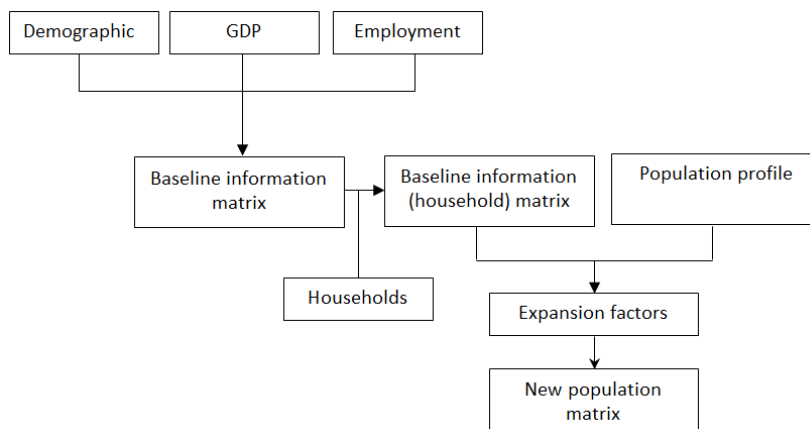
Car ownership is modelled by population segments, as defined in the population initial solution, based on information from both time series and cross sectional data referring to car ownership in Denmark. The data include information about household characteristics, car costs ownership

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<sup>27</sup> A tourough description of the NTM population syynthesis step is provided in Chaprter 7.

and maintenance and measures defining accessibility through alternative modes of transport. The car ownership model defines four classes of number of cars per household (0, 1, 2, 3 or more) and three car types (small, medium and large).

With respect to the population synthesis, the NTM strategic model uses a Prototypical Sample Enumeration (PSE) approach (Daly, 1998). Through the PSE it is possible to generate a synthetic representation of a population by expanding a set of baseline information to the population profile through expansion factors (Daly, 1998). In this way, the PSE creates a weighted version of the baseline information which is representative of the population (Rich 2011). In the NTM the PSE uses the Iterative Proportional Fitting (IPF) algorithm. The IPF works by iteratively fitting an initial solution that defines the structure of a population matrix, (i.e. the baseline information) and marginal totals which represent the (future) restrictions imposed on the population (i.e. the population profile). The overall population synthesis framework is graphically described by Figure 3-4.



**Figure 3-4. The NTM population synthesis framework.**

The population baseline information classifies the Danish population in segments homogeneous with respect to the variables relevant for the travel behaviour, such as type of household, location, income, etc. as follows:

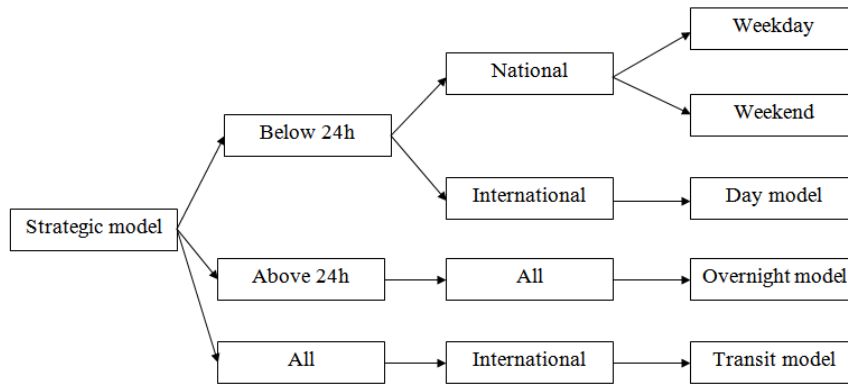
- household type (4 categories);
- residential zones (~1500 zones);
- region (2 regions);
- income category (4 categories);
- children (2 categories);

- age of the head-of-household (6 categories).

Based on the initial solution, the population synthesis step scales the travel demand to reflect the population profile and, in case of forecasts, their changes in over time.

### The passenger models

The research described in this thesis focused on the passenger demand model. The passenger models define transport individual choices as divided in five different segments, as graphically illustrated in Figure 3-5: national (week-day and weekend models), international (day model), national and international (over-night model) and international (transit model). The weekday model, which has by far the highest share of travel demand, is a random utility based framework. The overnight and international travel models are instead based on a linear regression model for generation coupled with a MNL model for mode/destination choice.



**Figure 3-5. NTM strategic and passenger models framework (Rich et al. 2010).**

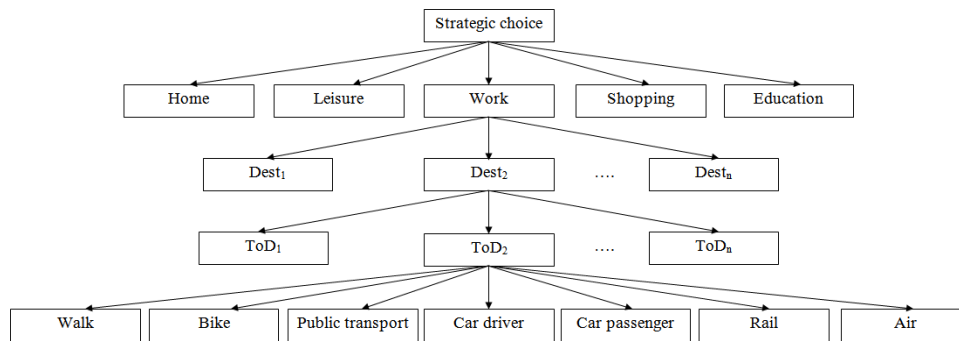
The analyses described in chapters 4,6 and 7, focused on the passenger national week-day model. The model is tour-based and the model structure can be divided in two main sub-models (Rich, 2010) modelling the primary tour activity of the day and the intermediate stop activities (conditional on the primary activity)<sup>28</sup>, according to observed chains in TU data. The model specification is a panel data with fixed time and group effects, as follows:

$$T_{nt} = \alpha_n + \gamma_n + \beta'_{nt} x_{nt} + \varepsilon_{nt} \quad (14)$$

<sup>28</sup> A limitation is imposed so that a tour can consist of a maximum of four trips (home-stop; stop-main destination; main destination-stop; stop-home) and only two tours are allowed per individual per day.

Where  $T_{nt}$  is the number of trips per person for a given type of individual  $n$  and time  $t$ ,  $\alpha_n$  and  $\gamma_n$  are the effects common for all individuals in the same group and for a specific time, and  $x_{nt}$  the vector of exogenous variables related to individuals, economy and level of service of the infrastructure.

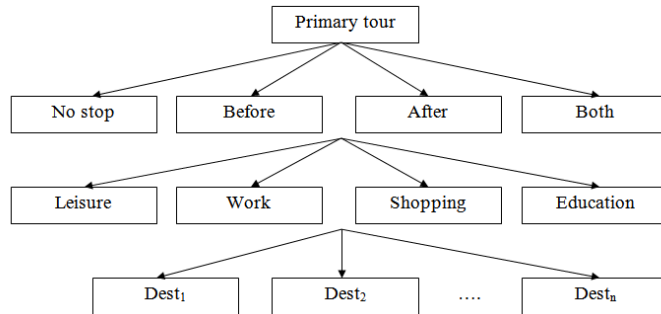
The modelling of the primary tour, graphically depicted in Figure 3-6, starts by identifying the primary activity (work, education, shopping, leisure and home). Socio-economic characteristics of the individual, in particular age group and labour market participation, are expected to be the main drivers of the individual choice among the alternatives. The primary activity determines the probability of each zone to be chosen as primary destination of the tour, depending on the trip purpose and the distance between the home and the destination. After that, the time of the day is modelled by calculating the combination of the start and the end of the tour. Finally, the mode of transport is defined. The primary tour defines the mode choice conditional on the time of the day and the main destination of the tour. The level of service variables and the socio-economic characteristics of individuals are the main determinants for mode choice.



**Figure 3-6. Choice model structure for the primary tour model (Rich 2010).**

Secondary activities are linked to the possibility of having intermediate stop activities. The model, graphically illustrated in Figure 3-7, defines if a secondary activity is chosen and when it is realised within the trip chain, i.e. if before or after reaching the destination for the primary activity or both. After that, the model delineates the purpose and the destination of the secondary activity trip. It is worth to highlight that to avoid inconsistencies, not all the combinations between primary and secondary activities are allowed and that the destinations of the secondary activities are conditional to the origin and destination locations of the primary tour. Both primary and secondary activities sub-models have a structure which hierarchically links the sequential choices, i.e. activity, destination, etc. Each choice is conditional to the one at

the level above and it is linked to the one at lower level through logsum variables that reflects the attractiveness of these lower level choices (Rich, 2010).



**Figure 3-7. Choice model structure for the intermediate tour model (Rich 2010).**

### Assignment models

In the NTM the assignment models share most of the features of the Næstved model. The main route choice models describe car and public transport, although the overall model also includes bike, walk, rail and air models. The road transport model is implemented based upon the travel matrixes output of the passenger and freight demand models, feeder transport to airports and preloaded bus routes (Rich et al., 2010). Park and ride is integrated into the modelling of transport chains, and the resulting car traffic is included in the road assignment model. The NTM assignment models explicitly model trips and derived congestion as a function of time of the day, thus implementing a dynamic assignment model. The public transport model is a combined schedule and frequency model, flexible in terms of calculation time and level of detail.

The NTM passenger assignment models solve by default for SUE by using MSA algorithm. The chosen route to travel by mode  $k$  between origin zone  $i$  and destination zone  $j$  is the one that minimizes the cost of travelling calculated at the link level as in the Næstved model:

$$C_{ijk} = \omega_l L_{ijk} + \omega_{fft} FFT_{ijk} + \omega_{tc} TC_{ijk} + \omega_c c_{ijk} + \varepsilon_{ijk} \quad (15)$$

The relation between travel time and traffic flows is based on the BPR formula.

### **The NTM forecast methodology**

The base year for the NTM is 2010, while the forecasts can be of any year for which the data can be provided (Rich et al. 2010). In case of forecasting, information is required about the future values of:

- population by gender, age and geographical distribution;
- national and regional GDP;
- employment by age and geographical distribution;
- income by age and geographical distribution;
- GDP and employment levels for EU foreign countries;
- fuel prices.

Part of this information feeds the NTM directly: Danish national and regional GDP, GDP and employment levels for EU foreign countries and fuel prices. Forecasts about population, employment and income levels are instead used as targets by the IPF algorithm in the PSE procedure. These forecasts, combined with population baseline information, generate new expansion factors that scale the baseline information to produce the forecasted synthetic population for the demand model, as graphically shown by Figure 3-4.

Data forecasts have different sources. The population targets correspond to the expected future population profile based on the forecasts from Statistics Denmark (SD), the Danish national statistics institute. The economic forecasts, both for GDP and employment levels, come instead from the ADAM model, which is a national model divided into 12 sectors implemented from the Danish Ministry of Finance. EU forecasts are used for foreign countries GDP and employment projections while the energy prices (fuel) forecasts come from the Danish Energy Authority.



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## **4 Paper 1: The effects of uncertainty in speed-flow curve parameters on a large-scale model: the Danish national model case study**

**Stefano Manzo, Otto Anker Nielsen, Carlo Giacomo Prato**

### **Abstract**

Uncertainty is inherent to transport models and prevents from using a deterministic approach when modelling traffic. Quantifying uncertainty thus becomes an indispensable step to produce more informative and reliable output of transport models. Within traffic assignment models, volume delay functions express the travel time as a function of traffic flows and theoretical capacity of the modelled facility. The US Bureau of Public Roads (BPR) formula is one of the most extensively applied volume delay functions in practice. This study investigated uncertainty in the BPR parameters. Initially, BPR parameters were estimated by analysing observed traffic data related to the Danish highway network. Then, BPR parameter distributions were generated by using re-sampling Bootstrap technique. Finally, the generated parameter vectors were used to implement sensitivity tests on the four-stage Danish national transport model. The results clearly highlight the importance for modelling purposes of taking into account BPR formula parameter uncertainty, expressed as a distribution of values rather than assumed point values. Indeed, the model output demonstrates a noticeable sensitivity to parameter uncertainty. This is particularly evident for stretches of the network with a high number of competing routes. Model sensitivity was also tested for BPR parameter uncertainty combined with link capacity uncertainty. The resulting increase in model sensitivity demonstrates even further the importance of the implementation of uncertainty analysis as part of a robust transport modelling process.

### **4.1 Introduction**

By modelling complex systems, transport models are subject to uncertainty that can affect all model components (i.e., context, model structure and methodology, inputs and parameters) to finally propagate to the model output. The main consequence of this inherent uncertainty is that transport models do not provide reliable point estimates of modelled traffic flows and derived measures. Instead, modelled traffic flows are better expressed as a central estimate and an overall range of uncertainty margins articulated in terms of (output) values and likelihood of occurrence (Boyce, 1999). Uncertainty analysis relates to how uncertainty in each model

component propagates to the model output and how to express the model output as a distribution, so reflecting the overall uncertainty present in the model.

The assignment algorithms of large-scale transport models often use static volume delay functions to express travel time as a function of traffic flow and theoretical capacity of the modelled facility. However, travel time is not just a function of flow and it is in fact affected by a number of different factors, such as downstream bottlenecks and resulting spillback or less than ideal weather conditions, causing drivers to drive slower. Consequently, a problem arises whenever traffic data output of static models are used to feed cost benefit analysis. In these cases, in order to produce valuable information, a necessary step is to address uncertainty in the volume delay functions by quantifying the sensitivity of the model output to the variability of the volume delay functions components.

Volume delay functions can be divided in three main groups (Akcelik, 1979): hyperbolic, polynomial and exponential. The US Bureau of Public Roads (BPR) formula, belonging to the polynomial group and proposed in its original version in 1964 (Bureau of Public Roads, 1964), is one of the most extensively applied volume delay functions in practice. The BPR formula, given free flow travel time, observed flow and link capacity uses parameters to represent different relationships between travel time and (modelled) flow-to-capacity ratios. Usually, the values for the parameters are pre-defined, based on assumptions and practice. However, as for any other model components, the BPR formula parameters have inherent uncertainty that originates from both the ignorance of the modeller of the true value of the parameters (epistemic uncertainty) and the stochastic behaviour of the (true) parameters itself (ontological uncertainty), which potentially vary by driver behaviour, time of the day, weather conditions and link characteristics.

An approach widely used in the transportation literature to quantify model uncertainty is to run model sensitivity tests by using distributions of input and parameters, and output of stochastic sampling procedures. For this purpose, re-sampling techniques such as Bootstrap (Efron and Tibshirani, 1993) have been used to generate model parameter distributions. Re-sampling approaches have a clear advantage compared to other sampling procedures. In fact, they do not require modellers' knowledge or assumptions about the shape of the parameter distributions, which becomes instead the output of the re-sampling methodology itself. Bootstrap has been implemented in many studies on transport uncertainty by Brundell-Freij (2000), Hugosson

(2005), Matas et al. (2012) and Petrik et al. (2012). Bootstrap defines the parameter distributions by recalibrating the model parameters for a number of model samples, which are generated from the original sample by re-sampling with replacement.

At the best of our knowledge, no attempt has been made so far to estimate uncertainty in the BPR formula parameters from the analysis of observed data and to analyse its effect on traffic assignment results of large-scale models. For this purpose, observations of the Danish highway network were obtained from the Hastrid dataset that is owned by the Danish Road Directorate. Non-linear regression analyses were implemented to allow the calibration of the values of the BPR formula parameters simultaneously. Afterwards, parameters were repeatedly calibrated on 10,000 Bootstrap samples to generate parameter distributions. Finally, selected percentiles of the distributions were used to run sensitivity tests on the Danish National Transport Model (NTM). In addition, a scenario investigating NTM sensitivity to BPR parameter uncertainty combined with link capacity uncertainty was tested. The link capacity uncertainty was quantified by creating vectors of capacity values through the implementation of Monte Carlo simulation.

The next section provides a description of the methodology applied to estimate the BPR parameter distributions, including a description of the datasets used for the research and the Bootstrap sampling technique. After a brief description of the NTM, the following section illustrates and discusses the results from the sensitivity tests run. The conclusions from this research are presented in the last section of this paper.

## 4.2 Methodology

### Time-Flow Relation: the BPR Formula

In traffic assignment models a common way to describe the relation between travel time and traffic flows is the BPR formula (Bureau of Public Roads, 1964):

$$TT_r = FFT_r * \left\{ 1 + \alpha * \left[ \frac{Flow_r + \gamma Flow_{r'}}{Capacity_r} \right]^\beta \right\} \quad (16)$$

where  $TT_r$  is the congested travel time on link  $r$ ,  $FFT_r$  is the free flow time on link  $r$ ,  $Flow_r$  is the traffic volume on link  $r$ ,  $Capacity_r$  is the capacity of link  $r$ ,  $Flow_{r'}$  refers to the traffic volume on the opposite direction of link  $r$  (relevant only in case of non-separated lanes), and  $\alpha$ ,  $\beta$  and  $\gamma$  are

volume-delay parameters. Specifically,  $\alpha$  represents the ratio between free flow speed and speed at capacity,  $\beta$  determines how steeply the curve bends once the capacity is reached and  $\gamma$  captures the effect of speed reduction due to opposite traffic in roads with non-separated lanes.

The BPR formula can be modified to express the relationship between speed (instead of congested time) and flow-to-capacity ratio, as illustrated by Nielsen and Jørgensen (2008) and Fagnant and Kockelman (2012):

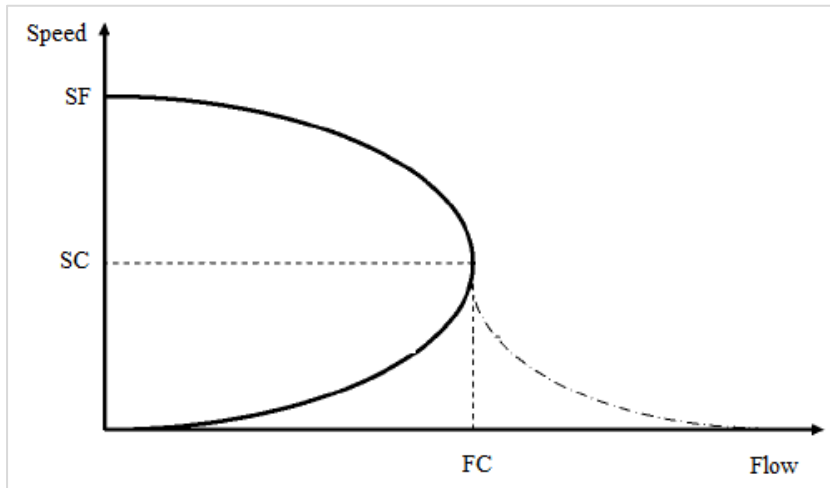
$$S_r = \frac{FFS_r}{1 + \alpha * \left[ \frac{Flow_r + \gamma (Flow_r')}{Capacity_r} \right]^\beta} \quad (17)$$

where  $S_r$  is the observed average speed on link  $r$  and  $FFS_r$  is the velocity in free flow conditions on link  $r$ . The use of either the time-flow or the transformed speed-flow formulas is generally data-driven, namely is dependent on the availability of data concerning either travel times or travel speeds. For example, the current study considers observations from a dataset of travel speeds, and hence uses the transformed speed-flow formula for the calibration of the BPR parameters. It is important to stress that the transformed formula implies an approximation. In fact, the speed is measured by local detectors, so it does not reflect precisely the link travel time, but rather is expression of the overall link conditions. On the top of our knowledge, no attempt has been done so far to quantify this discrepancy.

In general, criticisms have been moved to the BPR formula. As pointed out by Downing et al. (1998), depending on the choice of the parameter values the BPR formula may result insensitive to volume changes until demand exceeds capacity, when the predicted speed drops abruptly. Nevertheless, other studies proved that with an appropriate choice of parameter values specific for road type, the BPR formula offers comparable or even better goodness of fit to observed data than other volume delay functions (Klieman et al., 2011).

Another drawback is that the BPR formula results correct to model travel time only when the traffic flow is below capacity. In fact, when traffic flow reaches capacity (in Figure 4-1 the point corresponding to flow at capacity  $FC$  and the related speed at capacity  $SC$ ), the curve representing the BPR formula takes the shape of the dotted curve on the right of  $FC$ . Instead, the observed traffic behaviour is close to the pattern described by the bold line. To overcome this issue, it was suggested expressing the flow-capacity ratio in terms of density-density at

maximum flow ratio (Klieman et al., 2011). With this approach in fact, the speed-flow observations assume an s-shape that is possible to model.



**Figure 4-1. Assumed relation between speed and traffic flow.**

Despite the criticism, in static assignment models the BPR formula is commonly used and accepted for practical reasons. Among others, with the BPR formula the speed-flow relation curve is “continuous even beyond capacity and differentiable”, as argued by Nielsen and Jørgensen (2008).

### **Hastriid Dataset and Parameter Calibration**

This study intended to calibrate the BPR formula parameters, and hence used information regarding the Danish highway network that was contained in the dataset Hastriid, owned by the Danish Road Directorate. The Hastriid dataset contains observations for vehicle flow and average speed by time intervals of 15 minutes. The data used in the present analysis were collected in September 2009 from 3 count stations located in north east part of Zealand. Two count stations were located on the highway M11, called “Holbækmotorvejen”, connecting Holbæk, in the north-west part of Zealand, with the south-west suburbs of Copenhagen. The third count station was instead located on the highway M16, called “Hillerødmotorvejen”, connecting Hillerød, in the north part of Zealand, with the northern suburbs of Copenhagen. Table 4-1 summarizes the main characteristics of the three sections where the count stations were located while Figure 4-2 shows their geographical location on the highway network.

**Table 4-1. Characteristics of the Hastrid Dataset.**

Highway	Section	Section Length	Capacity	Lanes	Observations
Holbæk (M11)	Taastrup - Fløng	1.460 km	4200	3	1,141
Holbæk (M11)	Ringstedvej - Roskilde	0.953 km	3400	2	1,582
Hillerød (M16)	Farum - Skovbrynet	3.701 km	4200	2	1,229

NOTE: 1mi=1.61km.



**Figure 4-2. Sections location on the Danish (Zealand) highway network.**

In order to perform the parameter calibration, the 15 minute data were transformed into hourly data by summing the 15 minute vehicle flow observations and averaging the corresponding observed speeds. The flow-to-capacity ratio was calculated as density-density at a maximum flow ratio (Klieman et al. 2011). The density of maximum flow was defined at 28 passenger cars per kilometre per lane, corresponding to the value suggested by the Highway Capacity Manual (TRB, 2000) of 45 passenger cars per mile per lane. Finally, the free flow speed was calculated for each section as corresponding to the average observed speed at density-density at a maximum flow ratio lower than 0.5.

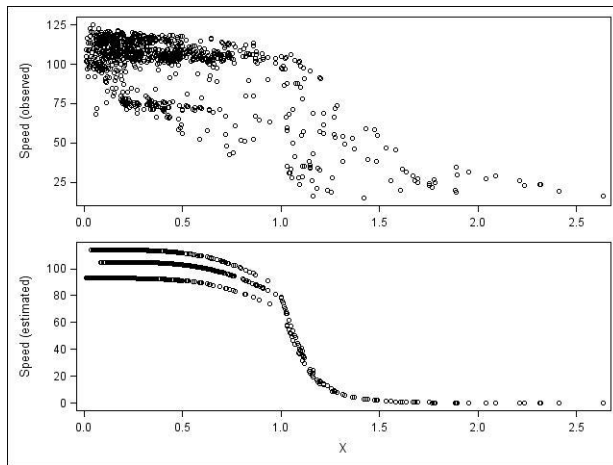
However, this approach may result in curves with a long tail on the right hand side (Hansen, 2010). This would imply the acceptance of relatively high speeds in situations over capacity, thus leading to an overestimation of the network accessibility. Thus, the density-density at the

maximum flow ratio approach was partially modified to better model severe congested conditions. Accordingly, for the calibration we used the value  $X$ , calculated as:

$$\begin{aligned} X &= D/D_{\max} \text{ if } D/D_{\max} < 1 \\ X &= 1 + 0.2 * (D/D_{\max}) \text{ if } D/D_{\max} \geq 1 \end{aligned} \quad (18)$$

where  $D/D_{\max}$  is the density-density at the maximum flow ratio. As can be seen, for severe congested conditions, i.e.  $D/D_{\max} \geq 1$ , the density-density at the maximum flow ratio values were reduced to avoid unreasonably high congested values.

The upper part of Figure 4-3 graphically shows the observed average speed plotted against  $X$ . Overall, the observed speed-flow relationship on the three links shows a trend consistent with what theoretically expected. As can be noticed, the majority of the observations cluster around the free flow speed of approximately 110km/h for low levels of congestion (corresponding to  $X < 1$ ). Only a few observations unexpectedly register free flow speed also in congested conditions (corresponding to  $X > 1$ ), probably due to count mistakes. Besides, there is a cluster of observations corresponding to speeds around 75km/h for low levels of congestion. These observations are probably related to trucks in the inner lane, which have speed limits of 80km/h (Nielsen and Jørgensen, 2008).



**Figure 4-3. Speed plotted against the density-density ratio.**

The parameter calibration, implemented using the statistical software SAS, resulted in  $\alpha = 0.33$  and  $\beta = 4.04$ . With respect to the Danish road network, Hansen (2010) defined a range of values between 0.5 and 2 for  $\alpha$  and between 1.4 and 11 for  $\beta$ . Thus, for validation purposes, vehicle speeds resulting from the BPR formula and the calibrated values of  $\alpha$  and  $\beta$  were calculated and



compared with observed average speeds through both regression analysis and visual inspection. Results from the regression analysis were satisfactory ( $R^2 = 0.9764$ ) as well as the ones from the visual inspection of the pattern of the speed estimated from the BPR formula, depicted in the bottom part of Figure 4-3.

#### **4.3 Quantification of Uncertainty in the BPR Formula Parameters**

In order to produce BPR parameter distributions, the re-sampling technique Bootstrap (Efron and Tibshirani, 1993) was used<sup>29</sup>. The Bootstrap method investigates the accuracy of an estimator  $\theta$  based on the assumption of considering the original sample, originating  $\theta$ , as the population. Bootstrap consists in a three step procedure. Firstly, from the original sample of  $n$  observations a number of samples are generated through (re)sampling with replacement. All Bootstrap samples contain  $n$  observations as the original sample. The replacement approach guarantees that each observation in the original sample has a constant probability  $1/n$  to be drawn; as a consequence the Bootstrap samples have a high probability of differing from each other. Secondly, the estimator  $\theta$  is calculated for each Bootstrap samples. Thirdly, the new  $\theta$  values obtained are analysed to infer the accuracy of the estimator by using some uncertainty measures such as variance or standard deviation.

One restriction to the use of Bootstrap is that it can be only implemented for variables which are the output of calibration processes and only when the sample is available. Thus, it cannot be applied to variables observed, assumed or imported. Besides, it is important to notice that the Bootstrap method has two downsides. Firstly, there is no rule defining the correct number of Bootstrap samples to generate, although the number should be large and, in theory, infinite. Secondly, the results are constrained by the quality of the original sample, given that the Bootstrap samples do not increase the amount of information there contained.

Using as original sample the one used for the parameter calibration, 9999 Bootstrap samples were created and the calibration process was repeatedly implemented for each of them. The resulting parameter statistics are summarized in Table 4-2. Also the Coefficients of Variation (CV) are reported and henceforward used as a measure of uncertainty. Table 4-2 also shows

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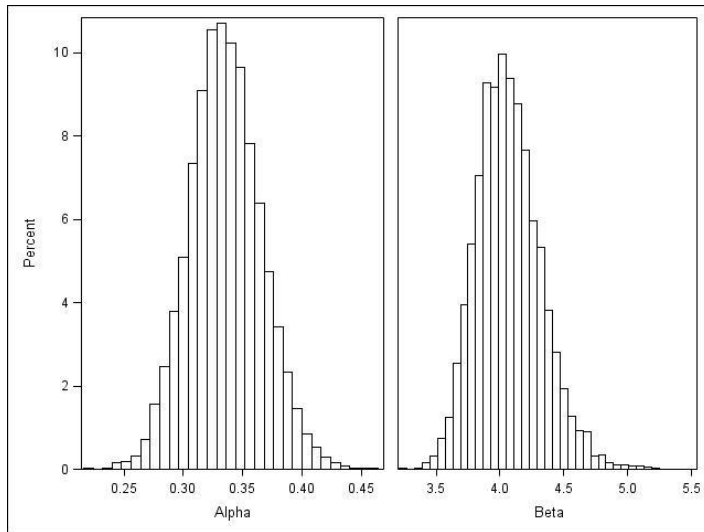
<sup>29</sup> An alternative approach would have been to use the standard deviations resulting from the regression analysis and assume a normal distribution. However, we preferred using Bootstrap method to avoid the assumption of normality.

selected percentiles of the distribution. The sensitivity tests on the NTM were run based on these values rather than for all 10,000 parameter values (9,999 from the Bootstrap samples plus one of the original calibration) because of the long run times of the NTM model<sup>30</sup>. Finally, Figure 4-4 graphically shows the resulting distributions for  $\alpha$  and  $\beta$ .

**Table 4-2. Bootstrap parameters statistics and distribution percentiles.**

Parameter statistics											
Parameter	Estimate	StDev	Min	Max	CV						
Alpha	0.335	0.030	0.216	0.462	0.090						
Beta	4.070	0.254	3.238	5.373	0.062						
Distribution percentiles											
Parameter	P1	P10	P20	P30	P40	P50	P60	P70	P80	P90	P99
Alpha	0,27	0,30	0,31	0,32	0,33	0,33	0,34	0,35	0,36	0,37	0,41
Beta	3,55	3,76	3,86	3,93	3,99	4,04	4,12	4,18	4,27	4,40	4,77

<sup>30</sup> Each sensitivity test was based on the output of 3 NTM runs, requiring a total running time of about 70 hours.



**Figure 4-4. Alpha and Beta distributions.**

### **Link Capacity Uncertainty**

Despite this study focuses on BPR parameter uncertainty, also the other variables of the BPR formula, namely  $FFT_r$  (or  $FFS_r$ ),  $Flow_r$  and  $Capacity_r$ , potentially have inherent uncertainty. A comprehensive analysis of the uncertainty deriving from the BPR formula should include also the assessment of model sensitivity to the uncertainty of these variables. However, with respect to NTM,  $FFT_r$  is based on legal speed limits and  $Flow_r$  depends upon trip generation processes, thus only uncertainty inherent to link capacity has been investigated.

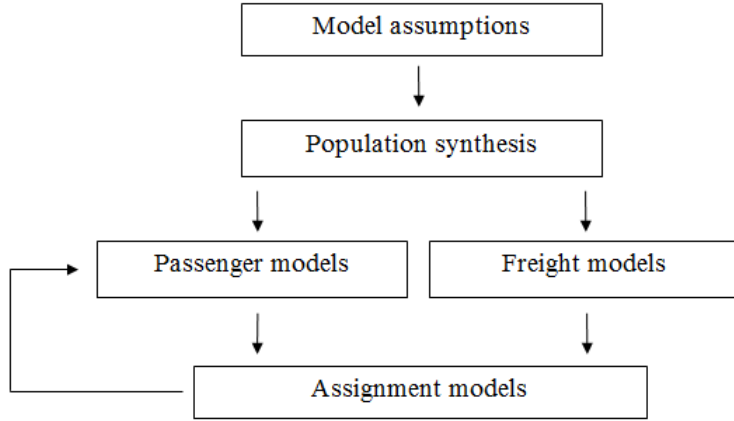
As previously highlighted, Bootstrap can only be applied to calibrated variables. Thus, Monte Carlo simulation has been implemented in order to quantify link capacity uncertainty. Triangular distributions were used in order to avoid illogical sampling results, such too high capacity values. The limits of the triangular distributions were defined as  $\pm 25\%$  of the capacity link value provided in the NTM network description. The resulting vector values were used in combination with BPR parameter values resulting from the Bootstrap procedure to run sensitivity tests on the NTM model. In this way it was possible to analyse the combined effect of the two uncertainty sources (i.e., BPR parameters and link capacity) on the model. As for the Bootstrap vectors, only selected percentiles from the Monte Carlo simulations were used to run the sensitivity tests.

#### **4.4 Case study: the NTM**

The NTM is meant to establish a unified reference model for transport policy analysis and project evaluation in Denmark (Rich et al., 2010). The model relies on two main data sources: the Danish travel survey, namely Transportvane Undersøgelsen (TU), and the Danish national register. TU is a national survey on-going from 1992 that contains travel information from around 1000 individuals per month, while the national register provides socioeconomic information for the entire Danish population. The model zone system is based on four different aggregation levels going from the more disaggregated up to the more aggregated: level 3 (regional level, 3670 zones), level 2 (national level, 907 zones), level 1 (strategic level, 176 zones) and level 0 (municipality level, 98 zones).

Figure 4-5 graphically describes the model framework. At the initial stage the model assumptions are defined, such as population, employment, and the road and transit networks. Based on this information, a prototypical population is created in the population synthesis step. Afterwards, the model consists of two parallel segments, the passenger demand model and the freight demand model. Both these models feed the assignment model that defines the route choice equilibrium. The equilibrium solution provides in turn feedback to the passenger demand models.

This study focuses on the passenger road assignment model. The model is tour-based and the model structure can be divided into two main sub-models modelling the primary tour activity of the day and the intermediate stop activities (conditional on the primary activity). A limitation is imposed so that a tour can consist of a maximum of four trips (i.e., home-stop; stop-main destination; main destination-stop; stop-home) and only two tours are allowed per individual per day.



**Figure 4-5. The NTM framework.**

More in detail, the passenger road assignment model is a link-based model solved by the Method of Successive Averages (MSA) to reach Stochastic User Equilibrium (SUE). The chosen route to travel by mode  $k$  between origin zone  $i$  and destination zone  $j$  is the one that minimizes the cost of travelling calculated at the link level as:

$$C_{ijk_r} = \omega_l L_{ijk_r} + \omega_{fft} FFT_{ijk_r} + \omega_{tc} TC_{ijk_r} + \omega_c c_{ijk_r} + \varepsilon_{ijk_r} \quad (19)$$

where  $C_{ijk_r}$  is the cost of travelling by mode  $k$  from zone  $i$  to zone  $j$  using link  $r$ ,  $L_{ijk_r}$  is the length of the link  $r$  by mode  $k$  from zone  $i$  to zone  $j$ ,  $FFT_{ijk_r}$  is the free flow travel time,  $TC_{ijk_r}$  is the extra travel time due to congestion,  $c_{ijk_r}$  represent monetary cost of travelling (varying according to mode and purpose),  $\varepsilon_{ijk_r}$  is the vector of residuals, and the  $\omega$ 's are the parameters associated to the respective variable. The relationship between travel time and traffic flows is based on the BPR formula.

## 4.5 Results and Discussion

The results from the sensitivity test runs on the NTM traffic assignment are summarized in Tables 4-3 and 4-4. The upper part of the tables (Scenario 1) shows results for model sensitivity to BPR parameter uncertainty. The bottom part (Scenario 2) illustrates instead results for model sensitivity to BPR parameter uncertainty and link capacity uncertainty combined.

Table 4-3 shows the links average CV referring to vehicle-kilometre (Veh-Km) and average speed (AvgSpeed) for both the entire network and the highway links only. As can be seen, the mean CV values for both Veh-Km and AvgSpeed are low, reflecting low model sensitivity to the BPR parameters uncertainty. However, it is worth to remind that uncertainty was quantified

only for parameters  $\alpha$  and  $\beta$  referring to highways links, which amount approximately to the 5% of the network. Besides, the parameter uncertainty resulting from the Bootstrap approach was high neither for  $\alpha$  (CV 0.09) nor for  $\beta$  (CV 0.054). As expected, the combined effect of BPR parameters uncertainty and links capacity uncertainty (scenario 2) increases the model uncertainty for both the overall network and the highways links.

The mean Veh-Km CV for highway links is lower than that for all links, despite the uncertainty was represented only in highway links. This comes as no surprise. In fact, for highway links the traffic demand can be assumed less elastic to changes in travel time (defined by the BPR formula) as compared to journeys using urban or local network. This assumption is primarily due to the lower number of competitive routes which characterizes journeys on highway facilities. Nevertheless, due to the differences in capacity, a small percentage variation in demand of traffic for highway links may easily result in a high variation for the links of the competitive routes that absorb the diverted traffic. This explains why the CV values for highway links result lower than for the overall network. With respect to AvgSpeed, the model appears to be insensitive. The reason can be probably traced in low congestion levels which characterize the overall network.

**Table 4-3. Veh-Km and AvgSpeed CV Statistics.**

		All links		Highway links	
		Veh-Km	AvgSpeed	Veh-Km	AvgSpeed
Scenario 1	Min	0	0	0	0
	Max	0.931	0.055	0.052	0.055
	Mean	0.011	0.000	0.003	0.001
	StDev	0.026	0.001	0.003	0.003
Scenario 2	Min	0	0	0	0
	Max	1.360	0.070	0.111	0.070
	Mean	0.015	0.001	0.012	0.007
	StDev	0.029	0.003	0.010	0.009

Table 4-4 shows the total network travel time, divided into free travel time and congested. As can be seen, the corresponding CV for both free and congested times is very low. This is

consistent and reflects the low variability resulting from the analysis of the AvgSpeed. However, links capacity uncertainty has a high impact on congested time uncertainty, which increases from 0.01 to 0.2.

**Table 4-4. Network travel time (Hours).**

		Mean	St Dev	CV
Scenario 1	Free time	17,727,618	18,012	0.001
	Cong time	935,988	9,738	0.010
Scenario 2	Free time	17,461,650	30,483	0.001
	Cong time	961,328	192,646	0.200

Despite overall the model showed low sensitivity to BPR parameter variation, the demand of traffic for some links revealed instead high elasticity, resulting in a maximum mean Veh-Km CV of 0.931 and 1.360 for Scenario 1 and Scenario 2, respectively. Thus, in order to analyse differences within the network, the data set was divided in three groups including links with Veh-Km CV lower than 0.1 (Group 1), between 0.1 and 0.5 (Group 2) and higher than 0.5 (Group 3). Statistics referring to the three groups are shown in Table 4-5.

**Table 4-5. Veh-Km CV by Groups.**

		Group 1	Group 2	Group 3
	Observations	33,385	307	25
Scenario 1	Min	0	0.100	0.501
	Max	0.099	0.494	0.931
	Mean	0.009	0.189	0.573
	StDev	0.010	0.089	0.110
	Observations	33265	442	10
Scenario 2	Min	0	0.100	0.507
	Max	0.099	0.481	1.360
	Mean	0.013	0.178	0.859
	StDev	0.013	0.088	0.392

As can be noticed, the majority of the links shows a modest or null sensitivity, consistently with the results for the overall model. Only a few links, included in the third group, show instead very high sensitivity, but because of their low number at least part of them are considered outliers. More interesting for modelling purposes are instead the links included in the second group. Most of them (around 200 in both scenarios) should be no cause for concern, given that they represent international Danish traffic and the relatively high variability is probably due to the low number of observations in absolute values. However, the remaining ones, for a total of 107 (scenario 1) and 241 (scenario 2) links, mainly refer to road types “hovedvej” and “trafikvej”, potentially hosting commuting traffic. As a consequence, the assessment of projects planned to be implemented in the areas of the network where they are located can be highly affected by their inherent uncertainty. In fact, in case of changes in the network due, for example, to structural changes or transport policy, the high sensitivity they demonstrated may cause the traffic to divert from the originally modelled routes. In areas characterized by a dense network, and hence many competitive routes, these changes can easily cause a shock wave throughout the surrounding network.

#### **4.6 Conclusions**

This paper describes the results of a study carried out to test the NTM sensitivity to BPR parameters ( $\alpha$  and  $\beta$ ) uncertainty. BPR parameter uncertainty was quantified using Bootstrap re-sampling approach. The speed and flow data used to calibrate the BPR parameters and, successively, to implement the Bootstrap analysis, refer to three highway links part of the Danish road network. Also model sensitivity to link capacity uncertainty, combined with BPR parameter uncertainty, was tested. The model output analysed were (i) vehicle-kilometre and average speed at the link level and (ii) travel resistance at network level.

The results confirm the importance of uncertainty analysis as a decision tool for transportation projects. In fact, although the NTM as a whole proved to be quite inelastic to the variability in the BPR formula parameters, some links showed high elasticity. Any assessment of projects potentially affecting traffic flow on those links should then take into consideration this elasticity and integrate uncertainty analysis in the decision process.

More in detail, the results clearly highlight the importance for modelling purposes of taking into account BPR formula parameter uncertainty, expressed as a distribution of values, rather than



assumed point values. The increasing amount of traffic data available nowadays, due to the diffusion and improvements of technology, allow in fact to estimate specific traffic delay formula parameters for different facilities and projects. This is an opportunity that should not be missed in order to produce more reliable modelled traffic results. Besides, when combined with uncertainty analysis, it may produce the necessary information required to increase the quality of the decision process and to develop robust or adaptive plans.

Limitations and avenues for further research should be acknowledged by this study. Firstly, a possible limitation relates to the limited amount of count stations providing the traffic data the analysis is based upon. Further research could use a higher number of count stations, with a wider geographical distribution, in order to calibrate parameter values more representative for the overall network. Nonetheless, the results clearly underline the importance of taking into account parameter uncertainty and their essence would likely not change but rather improve from additional data. Secondly, further analysis including urban and rural facilities parameters uncertainty would provide a more comprehensive picture on the topic, including the possibility of developing a class reference approach for uncertainty analyses of such kind. Lastly, due to the characteristics of the NTM and the scope of the study, the analysis presented in this paper did not quantify the effects on the model output deriving from uncertainty in the BPR formula variables free flow speed and link flows. Further research could investigate these issues, depending on the model tested and the objectives of the analysis.

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## **5 Paper 2: How uncertainty in input and parameters influences transport models output: a four-stage model case-study**

**Stefano Manzo, Otto Anker Nielsen, Carlo Giacomo Prato**

### **Abstract**

If not properly quantified, the uncertainty inherent to transport models makes analyses based on their output highly unreliable. This study investigated uncertainty in four-stage transport models by analysing a Danish case-study: the Næstved model. The model describes the demand of transport in the municipality of Næstved, located in the southern part of Zealand. The municipality has about 80,000 inhabitants and covers an area of around 681km<sup>2</sup>. The study was implemented by using Monte Carlo simulation and scenario analysis and it focused on how model input and parameter uncertainty affect the base-year model outputs uncertainty. More precisely, this study contributes to the existing literature on the topic by investigating the effects on model outputs uncertainty deriving from the use of (i) different probability distributions in the sampling process, (ii) different assignment algorithms, and (iii) different levels of network congestion. The choice of the probability distributions shows a low impact on the model output uncertainty, quantified in terms of coefficient of variation. Instead, with respect to the choice of different assignment algorithms, the link flow uncertainty, expressed in terms of coefficient of variation, resulting from stochastic user equilibrium and user equilibrium is, respectively, of 0.425 and 0.468. Finally, network congestion does not show a high effect on model output uncertainty at the network level. However, the final uncertainty of links with higher volume/capacity ratio showed a lower dispersion around the base uncertainty value. Results are also obtained from the implementation of the analysis on a real case involving the finalisation of a ring road around Næstved. Three different scenarios were tested. The resulting uncertainty in the travel time savings from the comparison of the three scenarios expressed in terms of coefficient of variation, turned out to be between 0.133 and 0.145, thus confirming the importance of uncertainty analysis in transport policy assessment.

## 5.1 Introduction

The literature on urban planning and transport planning has demonstrated that there is considerable inaccuracy between forecasted and observed traffic flows (e.g., Bain, 2003; Bain and Plantagie, 2004; Bain and Polakovic, 2005; Flyvbjerg, 2005; Flyvbjerg et al., 2006; Parthasarathi and Levinson, 2010; Welte and Odeck, 2011). The list of potential sources of such inaccuracy originates from the complexity of the systems generating traffic flows (van Zuylen et al., 1999). Complex systems are systems whose components interact in a way that is difficult to understand, thus making their output unpredictable. As a consequence, whenever a model is created to reproduce a complex system, its output will invariably be affected by uncertainty. Uncertainty pertains everything the modeller does not know to a full extent due to limited knowledge (e.g., statistical sampling) or stochasticity (e.g., parameter calibration) of some model components (Walker et al., 2003). Any of the model components can be affected by uncertainty: context, structure, inputs, parameters and final output.

The main consequence of such uncertainty is that the point estimates of modelled traffic flows, and their derived measures, only represent one of the possible outputs generated by the models. Instead, modelled traffic flows are better expressed as a central estimate and an overall range of uncertainty margins articulated in terms of output values and likelihood of occurrence (Boyce, 1999). In fact, analyses based on point estimates invariably produce unreliable results and decisions taken relying on them may easily lead to unexpected consequences. Thus, it is essential to assess transport model uncertainty by producing uncertainty measures. This can be done by investigating where the uncertainty originates, which are its main drivers and how it propagates throughout the model, especially for sequential iterative analytic frameworks such as the commonly used four-stage model.

Previous studies addressed uncertainty propagation throughout four-stage sequential transport model frameworks, such as Zhao and Kockelman (2002), Zhang et al. (2011) and Yang et al. (2013). They all found a common uncertainty propagation pattern, where uncertainty increases throughout the first three model steps, i.e. trip generation, trip distribution and mode choice, to finally reduce in the assignment model. Zhao and Kockelman (2002) argued that this reduction might be due to the network congestion effects on the trip assignment equilibrium procedure, implying that capacity constraints might reduce the variability of the results on the link flows. However, they also pointed out that the reduction of uncertainty in the assignment

step might also be the consequence of the accumulation on the same links of independent trips related to different origin-destination pairs. In their analysis of the Dutch national model, De Jong et al. (2007) found that congestion reduced final model output uncertainty but only to a minor degree. Zhang et al. (2011) investigated model output uncertainty for different levels of congestion. The results from their analysis showed that the higher the level of congestion, the lower the capacity of the assignment model to reduce the overall uncertainty. Rasouli and Timmermans (2013), when investigating uncertainty of origin-destination matrix tables using the Dutch national transport model “Albatross”, found that higher levels of traffic volumes (at zone level) result in lower levels of uncertainty for different model output. Thus, it can be said that there is no consensus on how network congestion affects final model uncertainty. Nevertheless, as pointed out by Ziemis et al. (2010), it is reasonable to expect that model output variability can be somehow sensitive to the level of congestion in the network.

The present study investigates uncertainty deriving from input (i.e. collected data) and parameters (i.e. calibrated parameters) in a four-stage transport model using Monte Carlo Simulation (MCS) performed by means of Latin Hypercube Sampling (LHS). The literature reviews by De Jong et al. (2007) and Rasouli and Timmermans (2012) showed that MCS is also used by Ashley (1980), Kroes (1996), Zhao and Kockelman (2002), Pradhan and Kockelman (2002), Krishnamurty and Kockelman (2003), De Jong et al. (2007) and Zhang et al. (2011). The MCS approach has also been used more recently, e.g. in Rasouli et al. (2012) and Rasouli and Timmermans (2013). However, none of these studies explored the uncertainty deriving from the choice of the probability distribution function to be used in the sampling procedure.

The current study contributes to the stream of the existing literature primarily by (i) investigating the impact on model uncertainty deriving from using different probability distributions in the sampling procedure, (ii) analysing the effect of assignment procedures leading to different equilibrium conditions, and (iii) examining uncertainty for different levels of congestion. The following section of this paper introduces the four-stage transport model used as case-study followed by a section that illustrates the methodology applied in this study. Results and conclusions are discussed in the last two sections of the paper.

## 5.2 Case study

The uncertainty analysis was implemented on the four-stage Næstved model. The four-stage transport model is an analytic framework that combines trip generation, trip distribution, mode choice and trip assignment (see, e.g., Ortuzar and Willumsen, 2011). Each model output is used as input for the model that follows, and the link flows from the trip assignment are used as feedback for the previous stages of the framework. The model is solved with an iterative procedure that concludes when the link flows reach equilibrium, which usually corresponds to the state of either deterministic User Equilibrium (UE) or Stochastic User Equilibrium (SUE) (see, e.g., Sheffi, 1985; Ortuzar and Willumsen, 2011). Given the wide use of the four-stage transport model framework, results from this study are straightforward to interpret and to compare with other literature and project results. The Næstved model describes the demand of transport in the municipality of Næstved, located in the southern part of Zealand. The municipality has about 80,000 inhabitants and covers an area of around 681km<sup>2</sup>. In the Næstved model, the area of interest is divided in 106 zones. The network, graphically described in the Figure 5-1, is composed by 315 links classified as “small”, “large” and “highway” which represent respectively around 92%, 5% and 3% of the number of links. The network contains all the roads present in the modelled area – including residential roads – and it is only roads in closed (dead-end) residential areas that are not coded, as well as very small rural side roads.



**Figure 5-1. The Næstved model network.**

Basically the modelled network consists of the city of Næstved, where there is congestion, and then a large uncongested hinterland. The traffic, modelled over a single 24 hour time interval, is divided in two modes, private and public transport, with the first absorbing around 85% of the demand, and in two categories, home/work and business. The model final output is based on 3 model's iterations which only involve trip distribution, mode choice and trip assignment stages; in other words, after the first model run the trip generation output is kept constant and is not influenced by the travel impedance of the network. In the Næstved model, the four stages are specified as follows.



### Trip generation

The trip generation stage uses cross-classification approach to calculate the number of trips produced and attracted by each zone. Trip production and trip attraction are specified respectively as:

$$P_i = \beta_{wp} WP_i + \beta_w W_i \quad (20)$$

$$A_j = \beta_{wpp} WPP_j + \beta_{wps} WPS_j \quad (21)$$

where  $P_i$  is the number of trips produced in zone  $i$ ,  $A_j$  is the number of trips attracted to zone  $j$ ,  $WP_i$  and  $W_i$  are the number of workplaces and workers in zone  $i$ ,  $WPP_j$  and  $WPS_j$  are the number of primary work places and secondary work places in zone  $j$ , and the respective  $\beta$ 's are the trip production and attraction rates, based on national statistics. To balance trip generated and attracted a balancing tool is then applied, as follows:

$$P_i = zP_{0i} + (1-z)P_{0i} \frac{\sum A_o}{\sum P_o} \quad (22)$$

$$A_i = (1-z)A_{0i} + zA_{0i} \frac{\sum P_o}{\sum A_o} \quad (23)$$

Where  $z$  is the balancing factor having values between 0 (production adjusted based on attraction) and 1 (attraction adjusted based on production). For the present study, the balancing tool was implemented with  $z$  having the value of 1.

### Trip distribution

The trip distribution stage is based on a double constrained gravity model:

$$\begin{aligned} T_{ij} &= P_i A_j a_i b_j f(c_{ij}) \\ s.t. \\ \sum_j T_{ij} &= P_i \\ \sum_i T_{ij} &= A_j \end{aligned} \quad (24)$$

where  $T_{ij}$  is the number of trips from zone  $i$  to zone  $j$ ,  $P_i$  is the number of trips produced in zone  $i$ ,  $A_j$  is the number of trips attracted to zone  $j$ ,  $a_i$  and  $b_j$  are balancing factors that ensure that both constraints are satisfied and  $f(c_{ij})$  represents a deterrence function calculated as follows:

$$f(c_{ij}) = c_{ij}^{-\eta} \exp(-\theta c_{ij}) \quad (25)$$

where  $c_{ij}$  represents the generalized cost of travelling from zone  $i$  to zone  $j$ , and  $\eta$  and  $\theta$  are parameters to be estimated. The Furness method is then applied to estimate trips  $T_{ij}$  given the deterrence function  $f(c_{ij})$ .

### Mode choice

The mode choice model stage is based on a binary multinomial logit model including two alternative modes: private (car) and public transport (bus). The utility function is specified as:

$$U_{ijk} = V_{ijk} + \varepsilon_{ijk} \quad (26)$$

where  $U_{ijk}$  is the utility of using mode  $k$  to travel from zone  $i$  to zone  $j$ ,  $V_{ijk}$  represents the deterministic component of the utility and  $\varepsilon_{ijk}$  represents the unobserved error. The probability of choosing mode  $k$  to travel from zone  $i$  to zone  $j$  is then given by:

$$P_{ijk} = \frac{\exp(V_{ijk})}{\sum_l \exp(V_{ijl})} \quad (27)$$

where  $P_{ijk}$  is the probability that a given mode  $k$  is chosen to make a trip from zone  $i$  to zone  $j$ . Finally,  $V_{ijk}$  depends on the on the generalized cost of travelling and it is specified as:

$$V_{ijk} = \delta_c c_{ijk} \quad (28)$$

where  $\delta_c$  is the calibrated parameter and  $c_{ijk}$  the generalized cost of travelling from zone  $i$  to zone  $j$  by mode  $k$ . Following the mode choice stage, the OD matrix from trip distribution is split in two matrixes for private and public transport, thus defining the number of trips by mode  $k$  from zone  $i$  to zone  $j$ .

### Trip assignment

For this study the assignment model is a link-based probit model solved by the Method of Successive Averages (MSA) to reach SUE. The chosen route to travel by mode  $k$  between zones  $i$

and  $j$  is the one that minimizes the cost of travelling, estimated at the link level and calculated as:

$$c_{ijk_r} = \omega_l L_{ijk_r} + \omega_{tf} TF_{ijk_r} + \omega_{tc} TC_{ijk_r} + \varepsilon_{ijk_r} \quad (29)$$

where  $c_{ijk_r}$  is the cost of travelling by mode  $k$  from zone  $i$  to zone  $j$  using link  $r$ ,  $L_{ijk_r}$  is the length of the link  $r$  by mode  $k$  from zone  $i$  to zone  $j$ ,  $TF_{ijk_r}$  is the free flow travel time,  $TC_{ijk_r}$  is the extra travel time due to congestion,  $\varepsilon_{ijk_r}$  is the vector of residuals, and the  $\omega$ 's are the calibrated parameters. The travel time/flow relationship is based on the Bureau of Public Roads (BPR) formula, which calculates the total travel time as the sum of free flow travel time and congested travel time:

$$T_r = TF_r \left[ 1 + \alpha \left( \frac{x_r + \gamma x_r'}{C_r} \right)^\beta \right] \quad (30)$$

where  $T_r$  is the total travel time on link  $r$ ,  $TF_r$  is the free flow time on link  $r$ ,  $x_r$  is the traffic volume on link  $r$ ,  $x_r'$  is the traffic volume on the opposite direction of link  $r$ ,  $C_r$  is the capacity of link  $r$ ,  $\alpha$  and  $\beta$  are the traffic/delay parameters, and  $\gamma$  represents the effect on speed reduction due to opposite traffic in non-separated lane roads.

### 5.3 Methodology

Correctly identifying the main sources of uncertainty eliminates, or at least reduces, the probability that new sources of uncertainty are discovered further in the modelling process. Given that in the present study no traffic flow forecasts are calculated and the geographical area is fairly small, the model context was assumed not to be affected by uncertainty. Similarly, the model structure uncertainty was not investigated. As a result, the uncertainty analysis focused on model inputs, parameters and, through propagation, output uncertainty.

The analysis was implemented through MCS. First, input and parameter vectors of 100 draws each were produced using LHS. In fact, LHS stratifies the probability distribution by dividing the cumulative curve into equal intervals and then taking one random value from each interval. As a consequence, it is possible to represent the distribution precisely with a lower number of draws.

The list of the inputs used in the Næstved model is shown in table 1, although the values specific for each zone are not shown because of the high number of data. Existing studies, such as Matas et al. (2012), showed how uncertainty in model inputs has a high impact on model forecast uncertainty. When forecasting, model input uncertainty is related to the future values of, for instance, population or fuel prices. Although the present study does not investigate uncertainty in forecasts, the uncertainty in model input was analysed as deriving from the data collection process. The information required to implement the LHS, was chosen as follows. The mean values are the ones observed for each zone/link. The distribution selected for the sampling procedure is triangular with limits defined as  $\pm 25\%$  of the observed values. The triangular distribution was preferred because of the defined (non asymptotic) bounds; this made it possible to avoid unrealistic high or low draws, especially considering that the modelled area is fairly small and the data collection system is reliable. With respect to correlation, values for primary and secondary work places and workers were drawn from a multivariate distribution with a correlation coefficient (observed) of +0.3. For the assignment model, the link lane capacity was drawn from a multivariate distribution with a correlation coefficient (assumed) of +1 between lane capacities forth and back, whilst free speed and queue speed were drawn from univariate distributions.

**Table 5-1.Næstved model inputs.**

Model	Correlation	Variable	Distribution
Trip generation	0.3	Work places	Triangular
		Workers	Triangular
	0.3	Work places primary	Triangular
		Work secondary places	Triangular
Trip assignment	0	Free speed	Triangular
		Queue speed	Triangular
	1	Lane capacity (For)	Triangular
		Lane capacity (Back)	Triangular

Table 2 summarises the main information on the LHS implemented on the model parameters, whose systematic variation is often ignored, especially in assignment and discrete choice models, as pointed out in Nielsen et al. (2002). The mean values are from the model whereas the StDev values are calculated according to the simulated CV of 0.3. With a few exceptions, all parameters were assumed to be lognormal distributed so as not to have draws with illogical

negative sign. This would in fact potentially generate counterintuitive model output results: for example, a negative  $\omega_{tc}$  value in the assignment model would imply a decrease in the cost of travelling  $c_{ijkr}$  resulting from an increase in the extra travel time due to congestion  $TC_{ijkr}$ . In the assignment model, the length parameter is not distributed. Finally, the  $\gamma$  parameter in the time-flow curve is equal to zero for highway infrastructure, where the separated lanes prevent the traffic in opposite direction from interfering with the driving speed, thus making it irrelevant for calculating the total travel time.

**Table 5-2. Næstved model parameters.**

Model	Category	Variable	Distribution	Mean	St Dev
Trip generation	Home/work	$\beta_{wp1}$	Lognormal	1.061	0.318
	Home/work	$\beta_{w1}$	Lognormal	1.432	0.430
	Home/work	$\beta_{wpp1}$	Lognormal	0.005	0.001
	Home/work	$\beta_{wps1}$	Lognormal	1.342	0.403
	Business	$\beta_{wp2}$	Lognormal	0.118	0.035
	Business	$\beta_{w2}$	Lognormal	0.159	0.048
	Business	$\beta_{wpp2}$	Lognormal	0.001	0.000
	Business	$\beta_{wps2}$	Lognormal	0.149	0.045
Trip distribution	Home/work	$\eta_1$	Lognormal	0.052	0.016
	Home/work	$\theta_1$	Lognormal	0.043	0.013
	Business	$\eta_2$	Lognormal	0.052	0.016
	Business	$\theta_2$	Lognormal	0.043	0.013
Mode choice	Home/work	$\delta_1$	Lognormal	0.060	0.018
	Business	$\delta_2$	Lognormal	0.060	0.018
Trip assignment (route choice)	Home/work	$\omega_{l1}$	No Dist	0.350	
	Home/work	$\omega_{tf1}$	Lognormal	0.520	0.156
	Home/work	$\omega_{tc1}$	Lognormal	0.820	0.246
	Business	$\omega_{l2}$	No Dist	0.350	
	Business	$\omega_{tf2}$	Lognormal	1.300	0.390
	Business	$\omega_{tc2}$	Lognormal	1.300	0.390
Trip assignment (BPR function )	Small road	$\alpha_s$	Lognormal	0.800	0.240
	Large road	$\alpha_l$	Lognormal	0.500	0.150
	Highway	$\alpha_h$	Lognormal	0.450	0.135
	Small road	$\beta_s$	Lognormal	1.500	0.450
	Large road	$\beta_l$	Lognormal	2.500	0.750
	Highway	$\beta_h$	Lognormal	4.000	1.200
	Small road	$\gamma_s$	Lognormal	0.150	0.045
	Large road	$\gamma_l$	Lognormal	0.100	0.030

Sensitivity analyses were implemented on the four different model stage outputs. Each calculation was based on 100 model runs (one run for each of the 100 input and parameters vector values). Uncertainty was expressed in terms of CV; although the amount and the meaning of each stage output are different, the average CV can be used for comparative purposes (Zhao and Kockelman, 2002). The Næstved road network is characterised by an average low level of congestion. Thus, to explore model output uncertainty in different traffic conditions, uncertainty was investigated under three different levels of generated traffic (i.e., GT 1.0, GT 1.5, GT 3.0, corresponding to the traffic generated by increasing 1.0, 1.5 and 3.0 times the default mean values of the socioeconomic input used in the generation model). The analysis focused on the assessment of model uncertainty deriving from the use of different distributions in the LHS, different assignment algorithms, and different levels of network congestion.

## **5.4 Results and discussion**

### **Variable distributions**

The choice of the distributions to be used in the sampling algorithm is a key step within the MCS. In fact, while the StDev defines the level of uncertainty, i.e. the spread around the mean, the distributions used in the sampling algorithm influence the likelihood of occurrence of the model outputs. For this reason, sensitivity analyses were run to see how the use of different distributions in the LHS affects the final model output. Two scenarios based on the assumption of all input and parameters being either normal or lognormal distributed were run. The resulting model output distributions, related to total number of trips, travel time and vehicle kilometres, were then compared with the reference case which, as previously described, used a mix of triangular and lognormal distributions. All three scenarios are based on the assumption of GT 1.5. Table 3 summarizes the key information for the three output distributions, including the most likely distribution according to the results from the Kolmogorov-Smirnoff (K-S) and  $\chi^2$  tests for continuous and discrete variables, respectively. As can be seen, the choice of the distributions does not seem to highly affect the final results, namely number of trips, travel time and vehicle kilometres, in terms of mean value and CV. However, as graphically represented in figures 2, 3 and 4, it does affect them in terms of output distribution; as a consequence, the probability of occurrence of single events and the cumulative probability in the tails of the distributions vary considerably. Furthermore, as in Zhao and Kockelman (2002) the results show a correspondence between the distributions used to implement the LHS and the resulting

model output distribution. Though the relatively low number of observations, 100 per distribution, somehow weakens the distribution fitting analysis, these results suggest that it may be possible to make a reasonably accurate prediction of the model output distribution based on the distributions used in the sampling procedure. In this case, to define the entire model output distribution a lower number of model runs would be necessary, i.e. enough to define mean and variance of the distribution. This would be an interesting finding due to the importance of reducing the computational burden of uncertainty analyses based on model sensitivity tests as highlighted, among others, by Rasouli and Timmermans (2013).

**Table 5-3. Variable distributions analysis results.**

Number of trips	Mean	StDev	CV	Median	Distribution test)	(X <sup>2</sup>
Mixed	1,926,504	371,886	0.193	1,897,351	Normal (13.740)	
Lognormal	1,925,083	361,227	0.188	1,883,180	Logistic (2.740)	
Normal	1,906,430	365,425	0.192	1,858,072	Normal (9.560)	
Travel time*	Mean	StDev	CV	Median	Distribution test)	(K-S
Mixed	3,000,999	552,876	0.184	2,942,967	Lognormal (0.065)	
Lognormal	2,971,228	499,507	0.168	2,914,192	Lognormal (0.059)	
Normal	3,000,531	595,556	0.198	2,922,134	Normal (0.061)	
Vehicle kilometre**	Mean	StDev	CV	Median	Distribution test)	(X <sup>2</sup>
Mixed	3,251,608	490,491	0.151	3,201,536	Lognormal (7.580)	
Lognormal	3,268,697	477,105	0.146	3,187,503	Lognormal (6.480)	
Normal	3,275,401	485,748	0.148	3,232,535	Normal (9.120)	

\*Minutes \*\*Thousands

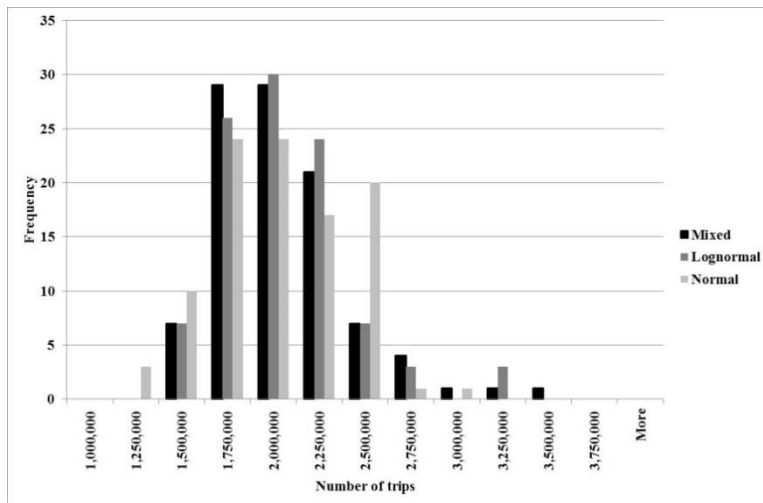


Figure 5-2. Total number of trips for different scenario distributions.

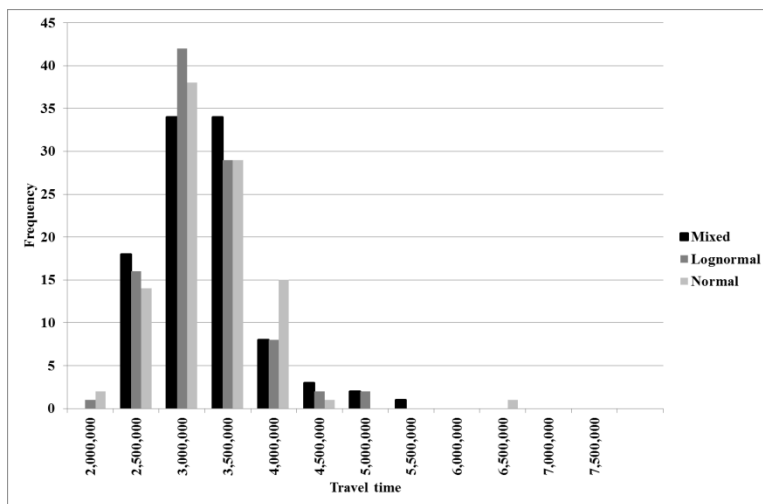
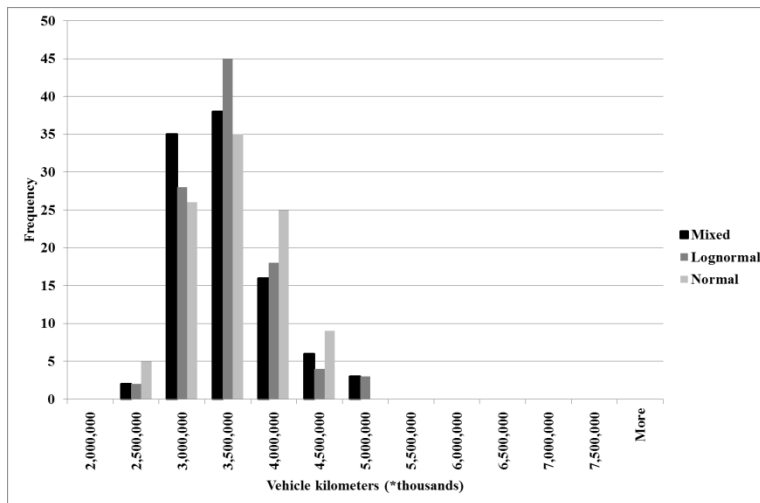


Figure 5-3. Travel time (minutes) for different scenario distributions.





**Figure 5-4. Vehicles kilometres (thousands) for different scenario distributions.**

### Assignment algorithms

Sensitivity analyses were run using MSA in the assignment algorithm to achieve, alternatively, SUE and UE at GT 1.5. As can be seen in table 4, with both assignment procedures the overall model uncertainty has an increasing propagation pattern throughout the first three model stages to then reduce in the assignment stage. This is consistent with what already found in existing literature: the assignment algorithm, by looking for network equilibrium, compounds and reduces uncertainty propagated from previous stages.

As table 4 shows, this study adds to the literature that the SUE approach reduces model uncertainty more than UE (i.e., to 0.425 as compared to 0.468). The stochastic network loading procedure in the iterative process of the SUE approach appears then to guarantee more stable results, unlike in Yang et al. (2013). It is worth to notice that the SUE approach implies stochastic noise irrespective from any assumed uncertainty. In order to decouple such noise, the model was run 100 times by assuming no uncertainty in the mean values of the parameters used in the stochastic network loading procedure. The resulting model output uncertainty was of CV 0.059.

**Table 5-4. Models output uncertainty for different assignment algorithms.**

	Generation (Tavel demand)	Distribution (O-D demand)	Mode (O-D mode)	Assignment (Link flow)
SUE	0.369	0.466	0.485	0.425
UE	0.369	0.466	0.486	0.468

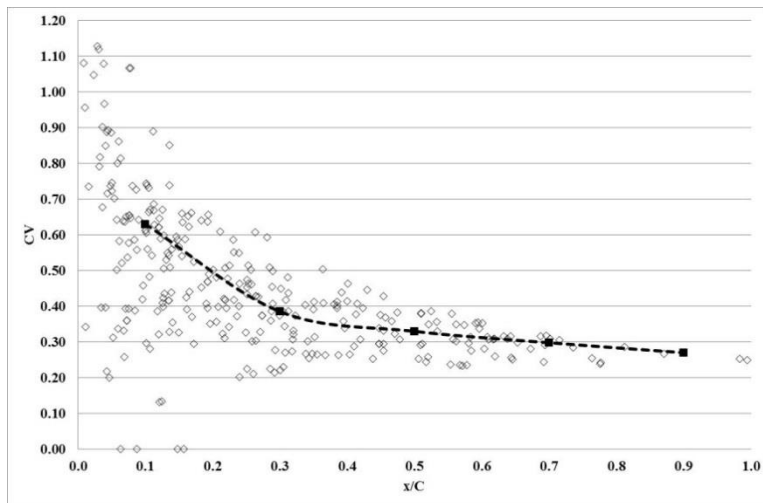
## Congestion scenarios

Nevertheless, often transport projects affect and focus on specific areas of the network rather than the entire network, so a link based analysis was implemented to produce further insight. The link based analysis was first implemented based on scenario GT 3.0 in order to investigate the network in the most congested condition. As graphically illustrated in the figure 5, the CV of links with higher volume/capacity ratio show a lower dispersion around the mean values (represented in the figure 5 by the squared markers, referring to  $0.2 \cdot x/C$  intervals, connected by the dashed line) converging towards CV 0.3. Besides, links with lower volume/capacity ratio are more likely to be associated with higher values of CV. The same analysis was implemented for GT 1 and 1.5 and produced similar results.

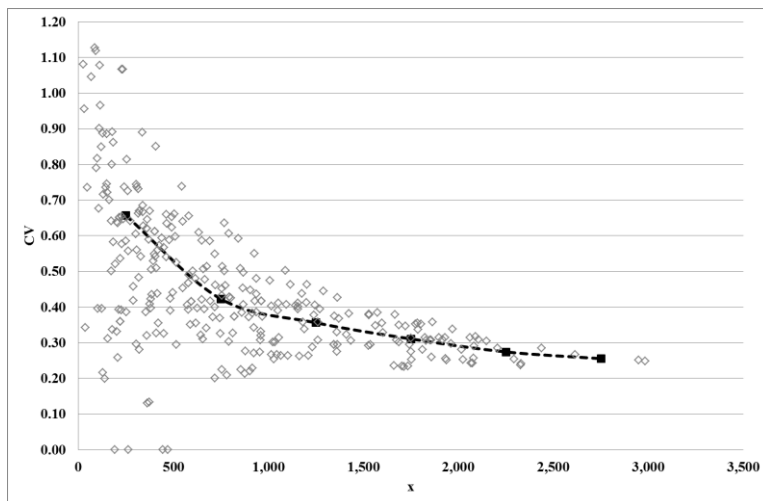
**Table 5-5. Models output uncertainty for different GT levels.**

	Generation (Travel demand)	Distribution (O-D demand)	Mode (O-D mode)	Assignment (Link flow)
GT 1	0.373	0.477	0.454	0.435
GT 1.5	0.369	0.466	0.485	0.425
GT 3	0.370	0.468	0.541	0.435

However it has been pointed out, for instance by Zhao and Kockelman (2002), that the increased levels of congestion might not be the source of the reduction in the levels of uncertainty. Such reduction could instead be ascribed to the presence on the same link of independent flows between various origin-destination pairs. For this reason the same link based analysis was implemented without considering link capacity but only link traffic volumes. As graphically shown in figure 6, by plotting CV versus traffic volumes, we obtained similar results as in figure 5.



**Figure 5-5. CV by link traffic volume ( $x$ ) and capacity ( $C$ ) ratio.**



**Figure 5-6. CV by link traffic volume ( $x$ ).**

The same analysis was performed with respect to link travel time. The majority of the travel time uncertainties lie between CV values of 0.35 and 0.40, irrespectively to the average link travel time. The low dispersion of the CV values is probably due to the low levels of overall link congestion; even in presence of variation in the number of vehicles in each link, this does not produce noticeable differences in travel time.

To summarize, the results from the present study showed low sensitivity of the model output uncertainty to the level of congestion in the overall network. However, the results from the link based analysis suggest that the volume/capacity ratio or, holding the assumption of

independency, the traffic volumes affect uncertainty in the modelled link flows but not in travel time.

### **Congestion scenarios: the Næstved ring road**

To further explore the effect of network congestion on model output uncertainty, an analysis was carried out to test the project of building a ring road around the city centre of Næstved. As previously said, the modelled network consists of the city of Næstved, where there is congestion, and then a large uncongested hinterland. In this respect, the ring road, graphically illustrated in Figure 5-7, is meant to relief the urban congestion thus reducing the overall travel time.



**Figure 5-7. Næstved ring road scenarios.**

The existing network already includes the west branch of the planned ring road, represented in black in Figure 5-7, while the east branch, in grey, is under construction. Given the base case, which implies the completion of the east branch, two scenarios were implemented, based on two different levels of network capacity (and corresponding level of service). The first scenario assumed the ring road fully completed, thus including also the north branch, in white in Figure 5-7, at the level of service of the existing west branch. The second scenario simulated the effects of an enhancement in the levels of service of the links part of the ring road, based on the

characteristics of the links in the network classified as highways. The scenario analysis assumptions are summarised in Table 5-6.

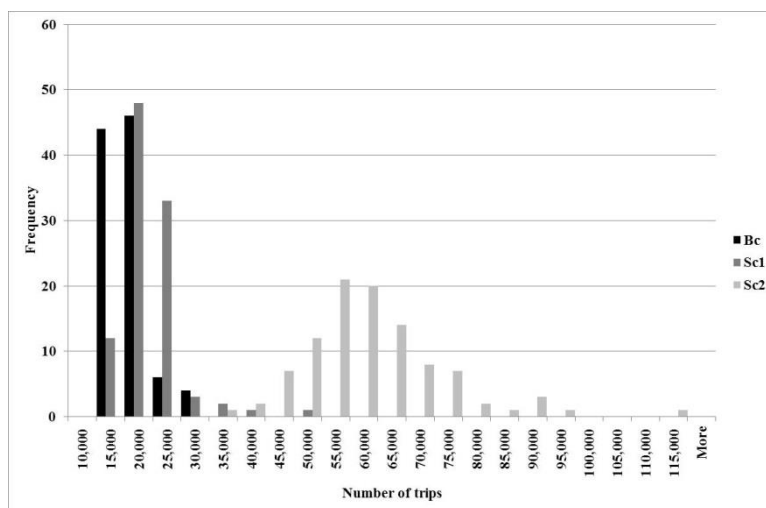
**Table 5-6. . Næstved ring road scenario analysis assumptions.**

	Free speed	Congested speed	Capacity
Base case	75	25	2000
Scenario 1	75	25	2000
Scenario 2	110	25	2200

For consistency and comparability the performed uncertainty analysis was based on GT 3 and, due to the aim of the ring road project itself, which focused on travel time savings. As shown in Table 5-7, the first scenario produced, on average, an increase of the travel time as compared to the base case caused by an increase of the average congestion in the network. In fact, the increased number of trips hosted by the ring road, graphically shown in Figure 5-8, results in a higher congestion in the access links. Given that both the capacity and the free speed in the ring road are unchanged, the infrastructure extension does not compensate for the loss of travel time and this causes an increase of network travel time. In the second scenario, the enhanced level of service of the ring road attracts even more traffic, as compared to the first scenario. This causes an increase in the level of congestion for the ring road and the access links but, at the same time, a decrease in the overall network average volume/capacity ratio. Besides, the increase in free speed in the ring road more than compensates the congestion level increase in both the ring road and access links. Consistently, results show substantial travel time savings as compared to the base case and the first scenario, in terms of both free travel time and congested travel time. However, the CV value for the congested travel time resulting from the second scenario is higher than that of the first scenario, thus implying a higher level of uncertainty related to the possible travel time saving output. This is of course to be taken into consideration given the importance of travel time savings as input for policy evaluation techniques, e.g. cost benefit analysis.

**Table 5-7. Scenario uncertainty analysis results.**

		Average	St Dev	CV
Base case	Free Time	3,328,632	518,113	0.156
	Congested Time	94,422	70,380	0.745
	Traffic volume/capacity	0.145		
Scenario 1	Free Time	3,670,441	551,689	0.150
	Congested Time	121,581	78,410	0.645
	Traffic volume/capacity	0.150		
Scenario 2	Free Time	3,278,724	507,546	0.155
	Congested Time	87,772	65,755	0.749
	Traffic volume/capacity	0.133		
Travel time savings	Bc -Sc1	-368,968	49,227	0.133
	Sc1-Sc2	425,525	61,705	0.145

**Figure 5-8. Number of trips in the ring road.**

## 5.5 Conclusions

This paper describes the methodology and results of a study focusing on uncertainty in transport demand modelling based on stochastic simulation combined with sensitivity analysis. The study focused on how uncertainty affects a four-stage transport model, with a case-study concentrating on the Danish town of Næstved. In particular, the analysis focused on: (i) investigating the impact on model uncertainty deriving from using different probability distributions in the sampling procedure, (ii) analysing the

effect of assignment procedures leading to different equilibrium conditions, and (iii) examining uncertainty for different levels of congestion.

The research highlighted that the impact of the choice of the variables distributions to be used in the sampling procedure is relevant in terms of model output distribution and thus requires specific attention. SUE and UE assignment algorithms were both tested and compared; results showed that the SUE approach reduces final uncertainty more than UE, due to the stochastic network loading procedure. The effects of different levels of congestion in the network were tested at the network level and at the link level. Results showed that although model output uncertainty is not sensitive to the level of congestion in the overall network, the higher the link congestion, or the link traffic volumes (holding the assumption of independency), the lower the dispersion of uncertainty around the mean values.

Given the importance of the output of transport demand models in policy evaluations, this study included a policy scenario analysis simulating three hypotheses related to the construction of a ring road around Næstved. In all scenarios, the values estimated for the travel time resulted to have a high level of uncertainty, in both absolute values and when comparing the scenarios in terms of travel time savings. This confirms how crucial uncertainty analysis is in order to produce a complete and informative assessment of any transport policy, given the key role that travel time (savings) has in transport policy project evaluations.

Further research on this topic is still needed. For instance a comparison of the results from different uncertainty analysis methodologies would prove useful to identify their advantages and disadvantages. Alternative approaches such as Bayesian melding (Sevcikova et al., 2007) could be considered for extending the scope of the research. Besides, long-term demand forecasts were not part of the present study. The uncertainty analysis of the demand forecast would imply addressing the issue of uncertainty in the model (system) context. This would lead to a combined methodology including scenario analysis for the model context uncertainty.

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## **6 Paper 3: Assessing the effects of uncertainty in socio-economic variables growth rate projections on a large-scale transport model forecasts**

**Stefano Manzo, Otto Anker Nielsen, Carlo Giacomo Prato**

### **Abstract**

A strategic task assigned to large-scale transport models is to forecast the demand for transport over long periods of time to assess transport projects. However, by modelling complex systems, transport models have an inherent uncertainty which increases over time. As a consequence, the longer the period forecasted, the less reliable is the forecasted model output. Describing uncertainty propagation patterns over time is therefore important in order to provide complete information to the decision makers.

Among the existing literature, only few studies analyse uncertainty propagation patterns over time, especially with respect to large-scale transport models. The study described in this paper contributes to fill the gap by investigating the effects of uncertainty in socio-economic variables growth rate projections on large-scale transport model forecasts, using the Danish National Transport Model as a case study. Population, gross domestic product, employment, and fuel prices, were analysed to quantify their uncertainty for 5 year intervals over a period of 15 years. The output of this procedure was then used to implement model sensitivity tests.

Results from the model sensitivity tests showed how the model output uncertainty grows over time, reflecting the increase in the uncertainty of the model variables. The resulting uncertainty temporal pattern was neither linear nor similar for the all the model output investigated. This highlights not only the importance of implementing uncertainty analysis specific for different model outputs, but also that a dynamic approach is required whenever the model has to provide mid-long time period forecasts.

### **6.1 Introduction**

Transport models are of great importance within transport project appraisals, since they provide insights into the demand responsiveness to changes in the transport system. This is true for all kind of appraisals, such as scenario-based forecasting studies, referring for instance to a national or regional masterplan, or more general supply oriented analyses, referring for instance to infrastructural changes. Usually transport projects have a medium-long run

perspective, which might easily go up to 30 years. This is not only because they require long time to be implemented, but also because transport demand needs time to adjust to changes on the supply side. Therefore, a key purpose of transport models, and in particular of large-scale models, is their ability to forecast the transport demand over medium to long time periods.

However, a reason of concern is the inherent uncertainty of the input variables to transport models. Uncertainty refers to any component of the system object of the modelling process that the modeller does not know to a full extent and, consequently, is not able to reproduce in the model with a deterministic approach. A specific issue is that the modeller's knowledge about the characteristics of the model components, such as context, inputs, etc., decreases the further the model forecasts are away from the present state. This is particularly true for the model variables that describe the external forces that produce changes in the reference system, such as the future values of the model socio-economic input variables. Consequently, it can be argued that model output uncertainty increases over time. For this reason, as pointed out by De Jong et al. (2007), defining the path of how model output uncertainty changes over time is of great importance. Allowing the inclusion of the levels of future uncertainty into the projects selection criteria would in fact guarantee a better comparison of alternative projects.

The rationale behind the present study is twofold. First, it aims to provide insight into how uncertainty in growth rate projections of socio-economic variables varies over time and into the effects of this variation on large-scale transport model forecasts. Secondly, it aims to outline a method to carry out such analysis by implementing a Monte Carlo Simulation (MCS) dynamic approach to compute and describe how uncertainty, represented in the MCS by the variables' Standard Deviation (StDev), varies over time.

The Danish National Transport Model (NTM) is used as a case study. The analysis focused on the uncertainty in the forecasted growth rates of the following socio-economic variables: population, Gross Domestic Product (GDP), employment and fuel prices. Uncertainty was quantified for 5 year intervals over a period of 15 years, producing StDev of the variables forecasts. MCS was then implemented using the official forecasted growth rates as mean values combined with the estimated StDev. The resulting 5 and 95 percentiles from the variable probability distributions were then used to run sensitivity tests on the NTM.

The following section 2 of this paper provides a literature review on the subject, while section 3 describes the NTM. Section 4 illustrates the methodology applied in this study. Results and conclusions are discussed in the last two sections of the paper.

## **6.2 Literature review**

The literature on uncertainty in transport models investigates both the sources and the effects of uncertainty in transport models; thorough reviews of the literature can be found, for instance, in De Jong et al. (2007) or Rasouli and Timmermans (2012). With respect to the model components, the literature investigated the uncertainty of the model input (i.e., the model exogenous variables), model parameters (i.e., the model calibrated parameters), or both. However, only a few papers focus on uncertainty deriving from the model input alone, such as Leurent (1996) and Rodier and Johnston (2002). Instead, the majority of the existing literature focused on both model input and parameters, as in Ashley (1980), Kroes (1996), Zhao and Kockelman (2002), Pradhan and Kockelman (2002), Krishnamurty and Kockelman (2003), Armoogum (2003), De Jong et al. (2007), Matas et al. (2011) and Zhang et al. (2011). Finally, some papers focused on model parameters uncertainty, such as Brundell-Freij (2000), Hugosson (2005) and Petrick et al. (2012).

With respect to the model input, such as the model socio-economic variables object of the present paper, the range of methodologies applied to investigate uncertainty include scenario analysis, such as in Rodier and Johnston (2002), and analytic expressions, such as in Leurent (1996). However, the majority of the papers investigated uncertainty through stochastic simulation, often MCS. Nevertheless, only a few papers investigated transport model uncertainty by quantifying the uncertainty propagation pattern over time. Rodier and Johnston (2002) implemented a scenario analysis on the travel demand and emission model of the Sacramento region (USA). They defined uncertainty margins for the variable forecasts through the study of existing forecasts and time series, and performed sensitivity tests for two years, 2005 and 2015. The results show an increase in uncertainty from 2005 to 2015 for all the model output analysed. Pradhan and Kockelman (2002) applied a MCS factorized design approach to quantify uncertainty in the land use variables of an integrated land use-transport model. The sensitivity tests, implemented over a 15 years period, show that model output uncertainty increases over the first 10 years to then reduce in the last 5 years, arguably due to model adaptation. Krishnamurty and Kockelman (2003) investigated uncertainty in the Austin

transportation model through MCS. They calculated uncertainty pattern over 15 years for peak and off peak vehicle hours and vehicle miles travelled. The uncertainty increases throughout the time period analysed. Matas et al. (2011) implemented an uncertainty analysis on traffic forecasts for the Spanish tolled motorway network over a 15 years period. Uncertainty was quantified through Bootstrap re-sampling method. Results show an increase on overall model uncertainty over the period. Thus, overall all the existing studies showed an increase over time of model output uncertainty. However, with the partial exception of Rodier and Johnston (2002), none of the aforementioned studies explicitly addressed the variation of uncertainty over time. In fact, the increase of model output uncertainty was related to the growth in the variables mean value over time and not in the variance, say, uncertainty, around these values, which was kept constant by the authors.

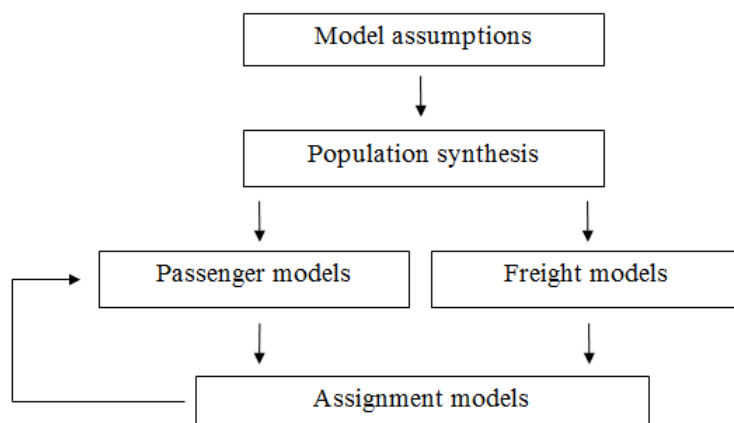
Another thing worth to be noted is that not many papers implemented their analyses by using large-scale transport models as case studies. The few exceptions include Hugusson (2005), which used the Swedish National Travel Demand Forecasting System, “Samper”, and De Jong et al. (2007) which run their analysis on the Dutch national model system, the Landelijk Model Systeem. Finally, as previously said, Matas et al. (2011) based their work on the Spanish tolled motorway network.

### **6.3 Case study – The Danish National Transport Model (NTM)**

NTM is a large scale transport model that has been developed for the Danish Ministry of Transport with the intention of providing a tool to be used for all transport project evaluations in Denmark (Rich et al. 2010) at both national and regional levels. NTM combines several sub-models, as graphically described in Figure 6-1. Preliminarily, the model exogenous variables, such as population, transport networks and employment, are defined. Afterwards, in the step called population synthesis, a population matrix is created through the forecasting methodology described later. Then, the framework divides in two parallel demand models: the passenger and the freight demand models. The output of these two models feeds the multimodal assignment models (including walk, bike, public transport, rail, car driver, car passenger and air), which is the last stage of the framework. The assignment models set the level of service per modes and routes by assigning traffic to the physical network at the link level. The level of service is then fed back to the passenger demand models, in an iterative process which ends when equilibrium between demand and assignment is achieved. Currently

this is accomplished through a heuristic approach based on a weighted method of successive averages. Overall, the model comprises more than 18 different sub models for different trip segments and durations and whether or not trips outside Denmark are included.

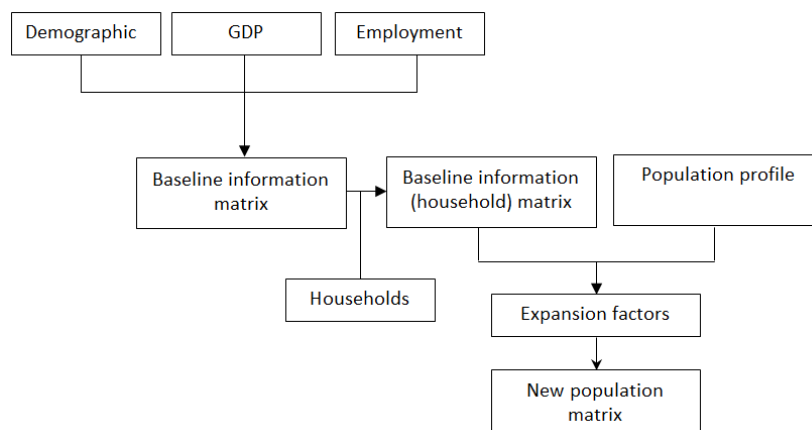
The passenger demand model is tour-based: the demand of transport is modelled as a sequence of trips, modelling the primary activity of the day and the intermediate stop activities (conditional on the primary activity), starting and ending in the same location. The model thus describes several trip purposes, the choice of trip frequency and destination. The traffic assignment model for car transport is a mixed probit multiclass stochastic user equilibrium model, and the public transport model is a schedule-based model. In the NTM, the zone system is based on four different aggregation levels, going from the more aggregated (municipality level, 98 zones) to the more disaggregated (regional level, 3670 zones). The road network consists of 34224 links.



**Figure 6-1. The Danish National Transport Model Framework.**

The forecasting methodology for the socio-economic variables in NTM is based on the Prototypical Sample Enumeration (PSE) approach (Daly, 1998) implemented through an Iterative Proportional Fitting (IPF) matrix estimation method. The PSE fits baseline information (e.g., resulting from a survey) to the population profile (target) created by using socio-economic forecasts or assumed scenarios. Eventually, the PSE creates a weighted version of the baseline information which is representative of the population profile (Rich, 2011). In the NTM, the population baseline information matrix combines three main datasets: demographic, GDP and employment which are then combined with the households information. The information comes from the Danish national register, which provides data regarding individuals, such as

employment status and age, households, such as number of children and income, and firms, such as number of employees and economic sectors. With respect to the population profile, the population forecasts are based on the forecasts from Statistics Denmark (the Danish Bureau of Statistics), while the economic profiles, based on GDP, employment and level of productivity by sector forecasts, are based on the forecasts of the Danish Ministry of Finance. The overall procedure is graphically described in Figure 6-2.



**Figure 6-2. NTM forecast framework.**

#### 6.4 Methodology for uncertainty analyses of input variables

Within uncertainty analyses it is common practice to select, for instance through preliminary sensitivity tests, the key model variables to investigate. The study described in this paper does not implement such selection. It focuses instead on the combined effect of the uncertainty deriving from the variables investigated, irrespective to the sensitivity the model shows to each of them separately. Furthermore, the analysis investigated the NTM socio-economic variables that have documented and available annual growth rate time series and forecasts, i.e. the population, employment, GDP (real) and fuel prices (both petrol and diesel). In particular, the focus of the analysis was on the uncertainty of the growth rate forecasts over a period of 15 years, from 2010, which corresponds to the NTM base year, to 2025. Data referring to the population, employment and GDP are used by the NTM in the PSE and, with respect to employment levels per zone, as zone attraction variable. The GDP values and the fuel prices, from the Danish oil industry association, are used to define, respectively, the value of time and the cost of travel per kilometre.



The uncertainty in the other NTM socio-economic variables, such as population spatial distribution, income distribution, work productivity and car ownership, was not explicitly investigated for different reasons. The analyses of the population spatial distribution and of the income distribution would require a scenario analysis approach which would not fit the stochastic Monte Carlo simulation analysis implemented for this study. The levels of work productivity depend on improvement in factors affecting production processes, such as technological innovation or more efficient corporate governance structures, which cannot be inferred from the observation of the past. In other words, according to the uncertainty taxonomy proposed by Walker et al. (2003), we are in condition of recognized ignorance. Therefore, to run the sensitivity tests the forecasted work productivity growth rates produced by the Danish Ministry of Economics were applied. Finally, car ownership (similarly to value of time) is estimated internally to the NTM, based on households characteristics, so its value reflects the uncertainty in the other socio-economic variables.

In order to quantify uncertainty in the variables' growth rate forecasts, multivariate normal Monte Carlo simulation was implemented by using Latin Hypercube sampling. In the Monte Carlo simulation, the choice of the distribution to be used in the sampling procedure is of crucial importance to correctly reflect the level of the variables' uncertainty. For the present study, the normal distribution was chosen for the following reasons. Firstly, given that in this study the variables investigated are annual growth rate forecasts, it was necessary to choose a distribution allowing representing both increases and decreases in the future values of the variables. Secondly, there was the necessity to choose a distribution symmetric around the mean and unbiased with respect to the possibility of drawing positive and negative values, given that we did not have prior expectations on that matter. Thirdly, the normal distribution allows reproducing a domain where values are not bounded between defined thresholds (due to the asymptotic tails of the distribution). Finally, the normal distribution emphasizes the likeliness of occurrence of the mean, and of the values around the mean, thus implying a degree of reliability of the forecasts, which we have no reason to doubt.

To implement the Latin Hypercube sampling, the official variables' annual growth rate forecasts for the years 2015, 2020, and 2025 were used as mean values. To describe the uncertainty pattern over time, the SD were produced for the years 2015, 2020 and 2015. Two different approaches were applied. With respect to the population, inspired by Rodier and Johnston

(2002), the SD were quantified based on the difference between the forecasts published in the Statistical yearbooks by Statistics Denmark from 1980 to 2005 and the observed population. First, the percentage difference of the population forecasts was calculated for each available 5, 10 and 15 year intervals. For instance, with respect to the forecasts published in 1980 for population in 1985 the percentage difference (PD) was estimated as follows:

$$PD_{1980/1985} = (\text{Forecast}_{1980,1985} - \text{Observed}_{1985}) / 100 \quad (31)$$

The resulting values are shown in the Table 6-1.

**Table 6-1. Danish population: resulting percentage differences between forecasted and observed values for 5 years intervals**

Forecasts publication year	Forecasted year				
	1985	1990	1995	2000	2010
1980	1.5%	2.1%	1.4%		
1985		-1.3%		-7.6%	
1990			-0.8%	-2.8%	
1995				-0.3%	-2.3%
2000					-0.7%
2005					-1.8%

These values, grouped for intervals of 5, 10 and 15 years, were then used to calculate the SD, shown in Table 6-2, used as a proxy for the population growth rates uncertainty in the Monte Carlo simulation. For instance, the SD for the 2015 population annual growth rate was calculated as follows:

$$SD_{Pop2015} = SD(PD_{1980/1985}, PD_{1985/1990}, PD_{1990/1995}, PD_{1995/2000}, PD_{2005/2010}) \quad (32)$$

With respect to the GDP, employment and fuel price growth rates, past forecasts were not available, so the SD were instead calculated based on the analysis of the annual growth rates time series. The method applied was the following. Having 2010 as NTM base year, the SD for the 2015 annual growth rate forecast was quantified based on the analysis of the annual growth rate time series referring to the period 2005-2010 (i.e. 5 years before the model base year). For 2020 and 2025 SD was quantified instead based on the time series referring, respectively, to the period 2000-2010 and 1995-2010 (i.e. 10 and 15 years before the model base year).

For instance, with respect to GDP, the SD to be used for 2015 was calculated based on the GDP annual growth rates for the period 2005-2010, as follows:

$$SD_{GDP2015} = SD(GDP_{2009/2010}, GDP_{2008/2009}, GDP_{2007/2008}, GDP_{2006/2007}, GDP_{2005/2006}) \quad (33)$$

This approach is meant to reflect a variation of the level of uncertainty throughout the forecasted period. Indeed, if the near future can be reasonably expected to be similar to the near past, the further in the future the model forecasts the broader is the range of events potentially occurring. These potential events require to be taken into consideration and, for this reason, events from a longer period in the past are included in the modelling process. For the present case study, this approach flattened or decreased the variability, expressed in terms of SD, of some of the forecasted variable values over time. This result is however expected, given the recent economic fluctuations which are foreseen to flatten in the near-mid future. As pointed out by De Jong et al. (2007), in the long run the economic variables might experience both periods of high and low growth, because of economic cycles. Thus, deriving SD from longer time series period tends to smooth the results. The mean values, i.e. the forecasted percentage growth, and the estimated SD of the variables used in the Latin Hypercube sampling are summarized in Table 6-2.

**Table 6-2. Inputs used to run the LHS on the NTM socio-economic variables.**

	2015		2020		2025	
	Mean	StDev	Mean	StDev	Mean	StDev
Population	0.3	1.2	0.3	2.5	0.4	4.5
GDP	1.8	3.5	133.7	2.5	1.0	2.3
Employment	0.6	2.7	0.5	1.9	0.1	1.8
Petrol	-1.2	5.9	0.9	5.2	0.6	6.0
Diesel	-1.3	11.0	1.0	9.3	0.7	9.7

The correlation coefficients used in the Latin Hypercube sampling for GDP, population, employment, petrol and diesel growth rates values sampling were estimated from the analysis of 30 years growth rates time series. Following a standard procedure, the variable correlations were tested for linearity, by comparing Spearman and Pearson correlation coefficients. The hypothesis of non-linearity was rejected and Pearson coefficients, summarized in Table 6-3, were then included in the analysis. However, only correlation coefficients between GDP and

employment (+0.859) and petrol and diesel prices (+0.774) were found significant at the 0.05 level and thus used to implement the Latin Hypercube sampling.

**Table 6-3. Pearson correlation coefficients of the NTM socio-economic variables.**

	GDP	Population	Employment	Petrol	Diesel
GDP	1				
Population	-0.345	1			
Employment	0.859*	-0.313	1		
Petrol	0.209		0.160	1	
Diesel	0.175		0.176	0.774*	1

\* Significant at 0.05

Finally, the multivariate normal Latin Hypercube sampling was run by using the mean values and the SDs from Table 6-2 and the correlation coefficients from Table 3. The p5 and p95 values of the distributions obtained from the Latin Hypercube sampling procedure, representing the annual growth rates for the selected years and shown in Table 6-4, were then used to run the sensitivity tests on the NTM along with the p50 which, as previously said, represents the variables' annual growth rate official forecasts.

One interpretative issue raises from this approach. For instance, the p95 model run simulates an increase in all the variable values. However, the effects of these values on the model output are of opposite sign. In fact, whilst a growth in population, employment and GDP is expected to increase the overall demand of transport, an increase in fuel prices, by increasing the cost of travel per kilometre, is of course expected to reduce it. However, the present study is interested in testing the overall effect of the uncertainty of these variables on the model output rather than decoupling their influence. Another reason of concern is that increases and decreases in oil prices, represented by increases and decreases in petrol and diesel prices, can reasonably be expected to have, respectively, negative and positive effects on the economy. However, economy needs time to adjust, thus this effects can expected to be observed in the following rather than in the very same year. This might also explain why the annual growth rate time series did not show significant correlation between economic variables, i.e. GDP and Employment, and petrol and diesel prices. However, to take into account this issue, selected scenario analyses were implemented, as described in the last part of this paper.

All the other variables used in the NTM such as, for instance, public transport fares and network design, were left unvaried. Afterwards, the results were compared with the 2010 NTM “base” output, as described in the following section.

**Table 6-4. p5 and p95 NTM socio-economic variable annual growth rates used to run the sensitivity tests.**

	2015		2020		2025	
	p5	p95	p5	p95	p5	p95
Population	-1.7	2.3	-3.7	4.4	-7.1	7.8
GDP	-3.0	6.5	-2.5	5.9	-2.7	4.7
Employment	-3.8	5.0	-3.0	4.0	-3.0	3.1
Petrol	-11.0	8.5	-7.6	9.4	-9.3	10.5
Diesel	-19.4	16.7	-14.4	16.3	-15.3	16.7

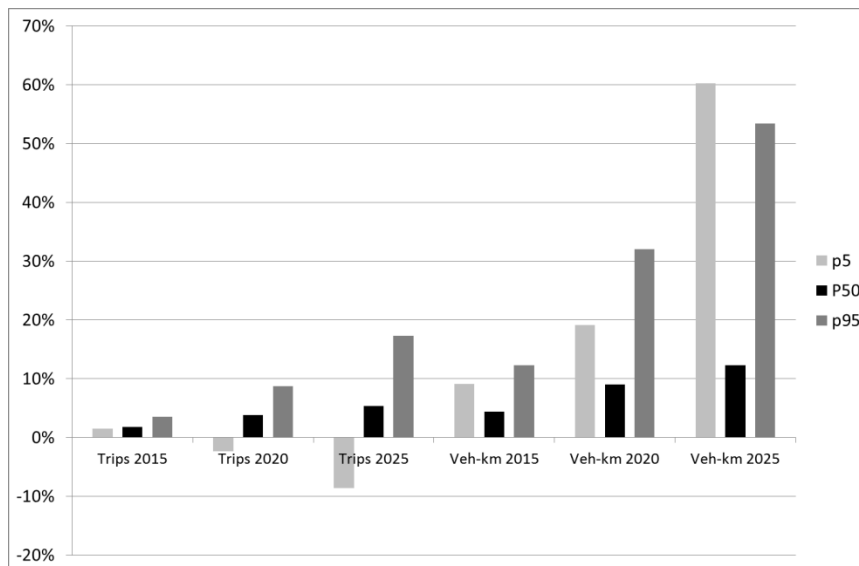
## 6.5 Results and discussion

The analysis was carried out while referring to the following transport modes: car driver (Car), car passenger (CarP), public transport (PT), bike and walk. The results from the sensitivity tests referring to the total number of trips for all modes and vehicle-kilometres (Veh-km) for motorized modes are summarized, in both absolute values and percentage change from the 2010 base case, in Table 6-5 and Figure 6-3. As can be seen, the increasing uncertainty over time, reflected in the increasing spread of the p5 and p95 of the variables distributions, results in increasing variability of the output results. For instance, with respect to the number of trips in 2015, the p5 output shows an increase lower than the corresponding p50, 1.55 as compared to 1.79, whilst the p95 output is 3.51% higher than the 2010 base case. The p5 2020 scenario produces instead a decrease in the number of trips of 2.39% as compared to the 2010 base case. In this case the decrease of fuel prices does not compensate for the decrease in population and GDP. Instead, p95 results for 2020 scenario produce an increase in the number of trips by 8.78% as compared to 2010, due to the increase in the population and GDP values which more than compensates the increase in the fuel prices. The p5 and p95 results for 2025 show an even bigger spread, with values of, respectively, -8.57% and 17.27%, due to the big difference in population growth rates between the p5 and p95 2025, of -7.1% as compared to 7.8%.

**Table 6-5. Sensitivity test results: Trips and Veh-km.**

Model runs	Trips*	Veh-km*	Trips**	Veh-km**
Base 2010	15,083	120,444		
P5 2015	15,317	131,400	1.55%	9.10%
P50 2015	15,354	125,746	1.79%	4.40%
P95 2015	15,613	135,227	3.51%	12.27%
P5 2020	14,722	143,515	-2.39%	19.16%
P50 2020	15,658	131,357	3.81%	9.06%
P95 2020	16,407	159,075	8.78%	32.07%
P5 2025	13,790	193,009	-8.57%	60.25%
P50 2025	15,898	135,202	5.40%	12.25%
P95 2025	17,687	184,803	17.27%	53.43%

\* Thousands \*\*Percentage change from Base 2010

**Figure 6-3. Trips and Veh-km percentage change from Base 2010.**

Unlike the number of trips, Veh-km p5 outputs show a higher increase than the corresponding base cases. For instance, in the p50 2015 scenario Veh-km is 4.4% higher than in the 2010 base case, whilst in the p5 2015 is 9.1% higher. This result reflects the increase in the car trip length following a decrease in travel cost per kilometre due to the reduced fuel prices. In fact, as can be seen in the Table 6-6, showing the average trip length by mode in percentage changes from the base 2010 scenario, while the average trip length for all modes reduces as compared to the base 2010 scenario, the length for the mode “car” increases. Furthermore, the p5 value for 2025 is higher than the p95 for the same year. Although counterintuitive, this result is

explained as well by the increase in the average car trip length, which is higher in p5 2025 (51.56%) than in p95 2025 (21.28%). This difference in car trip length is primarily due to the big difference in the fuel prices between the two model runs. In fact, whilst p5 2025 petrol and diesel values decrease by 9.3% and 15.3% respectively, p95 values increase by 10.5% and 16.7%, respectively. This also might explain the noticeable divergence between the effects on p5 2025 trips and Veh-km (-8.57% as compared to 60.25%). The difference in petrol and diesel prices between p5 2020 and 2025 can instead only partially explain the big difference between p5 2020 and 2025 Veh-km.

**Table 6-6. Average trip length by mode (percentage change from Base 2010).**

<b>Model runs</b>	<b>Walk</b>	<b>Bike</b>	<b>Car</b>	<b>CarP</b>	<b>PT</b>
P5 2015	-3.64%	-2.25%	3.66%	-1.03%	-3.41%
P50 2015	-3.08%	-1.71%	0.33%	-1.11%	-2.71%
P95 2015	-3.68%	-2.35%	3.95%	-1.24%	-3.08%
P5 2020	-7.49%	-4.69%	13.85%	0.08%	-0.03%
P50 2020	-5.71%	-3.15%	2.79%	-0.30%	0.73%
P95 2020	-7.23%	-4.82%	13.86%	-1.56%	-0.41%
P5 2025	-11.61%	-7.90%	51.56%	-1.98%	-3.13%
P50 2025	-6.99%	-4.04%	3.08%	-0.71%	-0.85%
P95 2025	-8.94%	-6.44%	21.28%	-2.62%	-3.26%

Table 6-7 shows the results from sensitivity tests related to the average speed (AvgSpeed), the free flow time (FreeT) and the congested flow time (CongT) calculated at the link level. As can be seen, with the network capacity held constant, the increase in the overall traffic over time reduces the average speed. This does not affect the free flow time, which remains substantially stable, but is instead reflected in the congested time, which shows high variability throughout the different model runs.

**Table 6-7. Sensitivity test results: Average speed, FreeT and CongT.**

<b>Model runs</b>	<b>AvgSpeed</b>	<b>FreeT*</b>	<b>CongT*</b>	<b>AvgSpeed**</b>	<b>FreeT**</b>	<b>CongT**</b>
Base 2010	53.94	53,652	5,321			
P5 2015	53.15	53,891	6,308	-1.46%	0.45%	18.56%
P50 2015	53.46	53,675	5,897	-0.90%	0.04%	10.82%
P95 2015	52.86	53,984	6,666	-2.00%	0.62%	25.28%
P5 2020	52.61	54,214	7,349	-2.46%	1.05%	38.12%
P50 2020	53.15	53,351	6,103	-1.47%	-0.56%	14.70%
P95 2020	51.54	54,356	8,484	-4.46%	1.31%	59.45%
P5 2025	50.22	55,595	12,231	-6.91%	3.62%	129.87%
P50 2025	52.82	53,231	6,487	-2.09%	-0.78%	21.92%
P95 2025	50.04	54,663	10,760	-7.24%	1.89%	102.22%

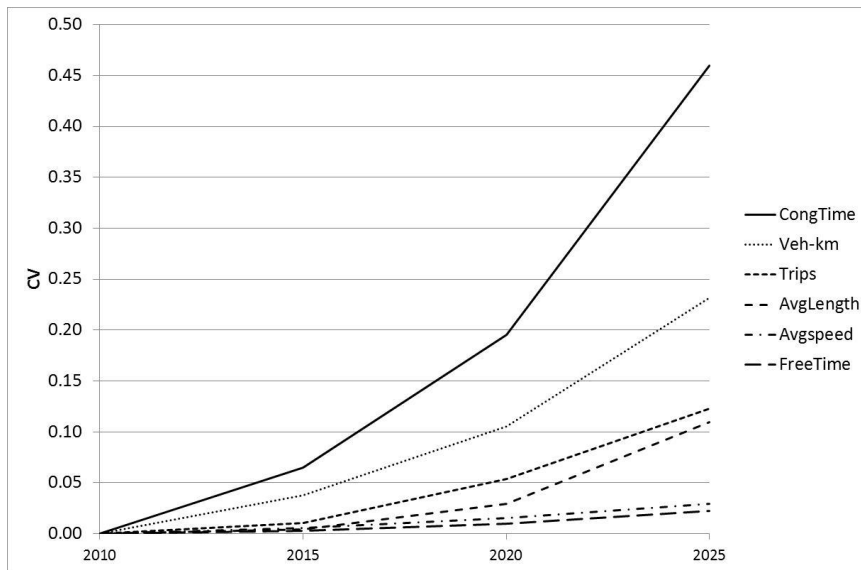
\* Thousands of hours \*\*Percentage change from Base 2010

Despite the limited amount of sensitivity tests produced, an attempt to infer the overall output uncertainty propagation over time was made by calculating the Coefficient of Variation (CV) for each of the outputs analysed above. The CV, corresponding to the StDev divided by the mean, is a measure commonly applied to quantify the level of uncertainty of a distributed variable. The results are summarised in Table 6-8 and graphically described in Figure 6-4. As can be seen, the CV increases over time for all the output investigated. However, while for the average speed and the free time this increase is scarcely noticeable, for other model output, and in particular for the congested flow time and the Veh-Km, the CV increase is clearly visible. This result is of great importance, considering the high relevance that these two model outputs have in transport project and policy appraisals. Furthermore, the applied methodology allowed to reproduce an increase over time of the output uncertainty, as can be seen by the non-linear propagation pattern over time. It is worth to notice that, as can be reminded from Table 6-2, not all the socio-economic variables investigated showed an increase in uncertainty over time, in fact only population and fuel prices. This suggests high sensitivity of the NTM to the population and fuel prices values.

**Table 6-8. Coefficient of variation by year and model output.**

<b>Year</b>	<b>Trips</b>	<b>Veh-km</b>	<b>AvgLength</b>	<b>AvgSpeed</b>	<b>FreeTime</b>	<b>CongTime</b>
2015	0.011	0.038	0.004	0.006	0.003	0.065
2020	0.054	0.106	0.029	0.015	0.010	0.195
2025	0.123	0.231	0.110	0.029	0.022	0.460





**Figure 6-4. CV propagation over time.**

In addition to the uncertainty propagation pattern over time, two 2025 socio-economic sensitivity tests were implemented, based on the results of the LHS summarised in Table 6-4: (1) low fuel prices (p5) combined with high GDP and employment levels (p95), and (2) high fuel prices (p95) combined with low GDP and employment levels (p5). While test (1) intends to simulate the effects of low petrol prices as potential driver for economic growth, test (2) instead is meant to reproduce the negative effects of high petrol prices on the national economy. Table 6-9 summarizes the total number of trips resulting from the implementation of the two sensitivity tests. The sensitivity test (1) produced, as compared to the p50 2025 base case, a modest increase in number of trips of 0.39%. With respect to the results from the sensitivity test (2), which was expected to reduce the demand of transport, the number of trips remains substantially stable as compared to the base case (0.05%). These results suggest low elasticity of the modelled number of trips to these variables. To investigate further this topic, two more sensitivity tests were then implemented representing, everything else staying constant at the base case levels, different population growth rates: (3) simulates an increase in the population (p95) as compared to the base case, whilst (4) a decrease (p5). As can be seen, the results from both model runs significantly differ from the base case. Indeed, as compared to the base case, the variation in the number of trips for sensitivity tests 3 and 4 is, respectively, of +11.08% and -13.41%. Thus, the model sensitivity to the variation of population growth rates is higher than that resulting from the combined variation of fuel prices, GDP and employment growth rates.

This seems to identify the population as the dominant variable affecting the model, among those examined.

**Table 6-9. Sensitivity analysis results (2025).**

	Base	Sensitivity test 1	Sensitivity test 2	Sensitivity test 3	Sensitivity test 4
Trips*	15,898	15,959	15,905	17,660	13,765

\* Thousands

## 6.6 Conclusions

The study described in this paper investigated the uncertainty in the NTM forecasts caused by the uncertainty of the forecasts of the model socio-economic variables, namely population, GDP, employment and fuel prices. The choice of using a large-scale transport model to run the analyses aimed to increase the amount of evidence on the topic related to large-scale models. In fact, despite their importance as support for strategic transport related decisions, there are not many studies investigating uncertainty analysis on large-scale models.

The analysis was carried out through stochastic simulation combined with model sensitivity analysis. The variables' growth rate forecast uncertainty was quantified through Monte Carlo simulation for 5 year intervals over a period of 15 years. A method to describe how uncertainty grows over time was implemented by computing SD for different time intervals of 5, 10 and 15 years. The rationale was to reflect the progressive decrease in the modeller's knowledge about the model components' future state by varying the SD to be used in the Monte Carlo simulation.

The SD were calculated based on the inaccuracies of past forecasts and past time series. This is a limitation in the sense that this approach allows to investigate only the component of the future uncertainty which is assumed to be rooted in the observed past variability. Besides, one source of uncertainty which should be addressed is the uncertainty variation over time of how individuals react to the different future values of the socio-economic variable. This is represented in the model by the calibrated parameters used, for instance, in the passenger and assignment models. On the top of our knowledge no attempt has been made so far to address such issue.

The model outputs analysed were (i) the total number of trips and Veh-Km, (ii) the trip average length by mode, and (iii) the average speed, free and congested time. The resulting temporal pattern of uncertainty was neither linear nor similar for the different model outputs

investigated. In particular, Veh-Km and congested time showed a higher increase in uncertainty over time.

Despite the results from the analysis described in this paper cannot be generalised, being related to a specific transport model, they nevertheless highlight two key points. First, they confirm the importance of implementing uncertainty analysis with a dynamic approach as part of a transport modelling process. In fact, different transport related projects may focus on different model outputs which have different temporal uncertainty propagation patterns. Thus, considering the long time horizon of transport project assessments, quantifying the uncertainty propagation pattern over time for key model outputs becomes strategically important. Second, the method suggested in this study to implement Monte Carlo simulation uncertainty analysis with a dynamic approach proved to be doable, so allowing such analysis to be conducted.

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## **7 Paper 4: When people move to the cities**

**Jeppe Rich, Stefano Manzo**

### **Abstract**

Urbanisation is currently a worldwide leading trend but the implication for the derived transport demand is not fully understood. On the one hand, agglomeration leads to spatial concentration which causes an increase in the number of generated trips due to the increase in the socio-economic opportunities for the population. Furthermore, urbanisation is not only a geographical movement but also a social transformation. In fact, it is the employed, richest, and younger to middle-aged part of the population, i.e. the part which generates more trips, that takes part in this migration. On the other hand, agglomeration generates significant re-bounce effects, i.e. a reduction in trip distances and transport demand due to higher levels of congestion. The net impact of these counteracting processes on the demand for transport is generally unknown.

In this paper, we investigate the derived transport impacts for three different population scenarios for Denmark in 2030. First, we consider the official 2030 baseline projection, here defined “spatial-social”, which is characterised by a strong underlying trend of people moving to the cities. Second, we consider a “naïve” forecast, where only the total size of the population is forecasted, but the spatial and social distribution is unchanged. Third, we develop a “social” scenario where we forecast the change in the social profile of the population while maintaining an unchanged spatial distribution. The different population profiles are constructed by using a detailed household population synthesis module, while demand impacts for the different populations are simulated by the Danish National Transport Model (NTM). Results show that the spatial inflow of people to the cities as compared to a naïve forecast leads to a reduction in the demand for transport per capita. The effect can be attributed to re-bounce effects, increased relative accessibility and density. The change in the social profile of the population has less impact.

### **7.1 Introduction**

Over the last decades, there has been a general and persistent worldwide trend of people moving to the major urban centres. The result of this urbanisation process is that today, more than half of the world’s population lives in urban areas and the share that is rapidly increasing.

Although the urbanisation is usually connected to developing countries it also exists in developed areas such as Europe and North America, Australia, New Zealand and Japan. Currently, over 75% of the population in these regions resides in urban areas and the share is expected to rise over the next decades (UN, 2011). One of the most noticeable results of this enduring urbanisation process in Europe has been the development of *functional urban regions* (Ratvez et al., 2013). This implies the integration of peripheral areas into the urban system, the connection of bordering cities to form polycentric networks and the creation of large-scale metropolitan regions (Nordregio, 2005). In Denmark the percentage of the population living in urban areas is currently above 80% (UN, 2011), with more than one third living in the major cities (Copenhagen, Aarhus, Odense, Aalborg and Esbjerg) according to the data from Statistics Denmark (SD). Over the period from 2006 to 2013 the population growth in the major Danish cities was higher than for the overall Danish population. More specifically, the urban area of Copenhagen experienced a double or even triple growth rate compared to the overall population growth (SD, 2013). The official forecasts produced by SD until 2040 show that this trend is expected to continue (SD, 2013). The reason why we observe this mega trend may be due to what has been referred to as *agglomeration effects* (Fujita et al., 1999). More recent empirical works by Rosenthal and Strange (2004) and Combes et al. (2012) reveal that the elasticity of work and firm productivity with respect to city size is positive and hence provide incentives for people and firms to move to the cities.

Not only there is a spatial flow from rural areas to urban areas, but there is also a heterogeneous social flow in the sense that it is the employed, richest, and younger to middle-aged part of the population that takes part in this migration. This trend has been observed in the last couple of decades at a global level and occurs in Denmark as well. In essence, it is the most active and wealthiest part of the population with the propensity to higher car ownership, longer and more frequent trips, which moves towards urban areas. However, the outcome in terms of demand of transport resulting from these joint processes is not clear.

Urbanisation affects demand of transport in many different ways. For instance, it has effects on number of trips and distances travelled, due to the increased proximity of trip origins and destinations, and also on the mode choice, due to the variation on the Level of Service (LoS) related to different modes. Besides, the social mobility deriving from urbanization, i.e. the potential overall enrichment of the population, also contributes in shaping the demand of

transport. However, whilst the consequences on demand of transport deriving from variations on LoS changes have been thoroughly investigated, those produced by residential and job locations or socio-economic characteristics of the population have not been yet. In particular what seems to be missed so far is the understanding of how these phenomena interact. As a matter of fact, if on the one hand when people move to denser areas they are expected to reduce the travelled distances per trip, on the other hand they may increase the number of trips due to the increased social opportunities. Besides, if they also experience an increase in income, this might further increase the overall demand of transport. Furthermore, there is the rebound effect due to the increased congestion levels, which will reduce the mileage travelled due to increasing costs.

In the existing literature, the effects of urbanisation on demand of transport have been mainly investigated in terms of mode choice, car-ownership and trip distances. A key factor in shaping the demand of transport is the urban density and land use. Bento et al. (2005) show that population density has a significant effect on car ownership in the sense that households located in less sprawled cities are less likely to own one or more vehicles. Scheiner (2006) makes a review of results from German case studies on how travel distance and mode of transport depend on spatial structures. What is shown is that inhabitants of dense, compact cities generally covers smaller travel distances and that the motorisation rate is usually lower than for inhabitants with similar socio-economic characteristics living in suburban and rural areas. Similar results have been found in other studies, such as Camagni et al. (2002), Sultana and Weber (2007) and Boussauw et al. (2012), referring to different urban areas. At macro level, in their comparative work on travel trends in 8 industrialized countries, Millard-Ball and Shipper (2010) also argue that lower demand of transport, expressed in passenger km per year of motorized travel, may be associated to countries with higher density and shorter potential travel distances. Scheiner (2010) highlights how mixed urban structures, say urban structures characterised by mixed land uses and complementary functions, allow for shorter trips, which can be carried out by non-motorised transport modes or public transport. This is also due to rebound effects from higher cost of travelling by car. Hankley and Marshall (2010) investigate US scenarios. Find that compact growth will reduce mileage and carbon emissions.

Another important factor affecting the demand of transport is the employment/labour force ratio. Schwanen et al. (2004) examines the effects of the urban structure on the commute



behaviour in the Netherlands. They show that employment/labour force ratio is negatively correlated with the probability of commuting by car. When more jobs are available for workers, suitable employment may be easier to find relatively close to home. Other modes of transport, such as bicycle, then become more attractive. Similarly, Bento et al. (2005) demonstrate that an increase in population density and work-housing balance<sup>31</sup> increase the probability of walking or biking to work.

Also person and household specific socio-economic characteristics are known to affect demand of transport. Schwanen et al. (2004) observe that the probability of driving to work by car increases as the level of car ownership and/or personal income increases. However, more highly educated workers are less likely to commute by car; this finding may reflect the fact that many highly educated people both live and work in more urbanized areas. Bento et al. (2005) show that higher income workers are less likely to walk or take public transport to go to work than they are to drive, whilst level of education has the opposite effect.

These relations between urban density, structure and socio-economic characteristics of the population on the one hand and demand of transport on the other hand, are of extreme relevance in terms of exploring the ways by which travel distances can be reduced. In fact, as pointed out by Bannister (2011), reducing travel distances may cause a decrease in travel times and speed, which in turn would result in gains in terms of travel time savings and reduction of energy consumption and pollution generation.

All of the references mentioned above apply either a descriptive or econometric approach in order to connect various spatial or urban attributes to transport behaviour such as mode choice or car ownership. However, none of the references simulate what happens if the estimated or observed behaviour is applied to a thoroughly constructed future population, which is forecasted at the social and spatial level. The contribution of this paper is to carry out a detailed population forecast which is anchored in official forecasts at the spatial and social level and then simulate the various population layouts in a transport model for the entire Denmark. The forecasts are based on official population forecasts from SD while income and employment forecasts are based on the Danish National Macro Economic Model from the Ministry of Finance, called ADAM (SD, 2013).

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<sup>31</sup> The work-housing balance is measured in terms of how evenly jobs locations are distributed relative to housing locations.

In particular the present study has two focuses. The first is on the trade-off between increased demand of transport, due to a younger and richer population, and the rebound effect, which is expected to reduce the mileage because of the increasing cost of transport due to congestion. The second is on the effects on demand of transport of changes in job location. To enable a detailed investigation on how movements in geographical and socio-demographic space affect transport behaviour, three scenarios are tested:

- 1) Naïve, where only the total population changes but with unchanged spatial (relatively speaking) and socio-economic profile compared to the 2010 baseline;
- 2) Socio, where the total of the population and the socio-economic profile is allowed to change but where the spatial pattern is unchanged;
- 3) Spatial-social, where individuals are allowed to move in the geographical space and in the socio-economic space.

The structure of the paper is as follows. Section 2 describes the NTM structure, focusing on the forecasts procedure and the methodology applied to create the scenarios used in the sensitivity tests. Section 3 presents the results from the sensitivity tests while section 4 includes a discussion on the results. Finally, conclusions from this study are illustrated in the last section of the paper.

## **7.2 The population synthesis and the transport model**

Methodologically the analysis in this paper is based on two parts. In the first part, we create the different population profiles by utilising a state-of-the-art household population synthesis model. The population synthesis model includes in itself two stages. First, a population fitting stage where future populations are fitted according to detailed constraints representing the future profile of the population. Second, a simulation stage where individuals are grouped into households at the micro level. The second part reveals future transport impacts of the different population profiles but running the NTM. In addition to the assumptions about the population, the NTM is also based on assumptions about infrastructure, employment, prices and the economic development. These assumptions are the same for all NTM runs.

### **The population synthesis**

Generally, the demand models operate on a list of individuals linked into households in order to be able to model household decisions such as car ownership. As a result, the two main tasks of

the population synthesiser are (i) to predict the number of individuals within a detailed socio-group conditional on future targets, and (ii) to consistently link these individuals in household entities. We will refer to the first task as the “population fitting” and the last task as the “household simulation stage”. If the model is run on a sample rather than the entire population (5.4 million individuals grouped into 2.6 million households, 2010 baseline) the corresponding demand is then up-scaled to the level of the population. When applying say a 10% sample, the population is sampled using sampling quotes in order to match the correct proportions at the level of municipalities and age classes. Re-scaling also applies after the household simulation stage in order to ensure consistency between the population fitting and the final list of individuals grouped into households.

### The population fitting

The objective of the population fitting stage is to construct a forecasted “master table” for the population. Table 7-1 below shows the resolution level at the spatial geographical and socio-economic level.

**Table 7-1. Elements and dimensionality of the master table for individuals.**

Dimension	Elements	Comment	Index reference
Municipality	98	L <sub>0</sub> zone system	L <sub>0</sub>
Residential zone	907	L <sub>2</sub> zone system	L <sub>2</sub>
Children	2		c
Age group	10		a
Gender	2		g
Labour market association	6		l
Personal income	11		i
Single	2		s
Cell combinations	5,280		
×L <sub>0</sub>	517,440		
×L <sub>2</sub>	4,788,960		

At spatial level there are 98 municipalities, which are divided in 907 Danish zones<sup>32</sup>. These are referred to as the L<sub>0</sub> and L<sub>2</sub> zone-system<sup>33</sup>. At the socio-economic level, for every L<sub>2</sub> zone there is a total of 5280 socio groups spanned by age, income, gender, labour market association and whereas the individual have children or are from a single adult household. This leads to slightly

<sup>32</sup> The foreign zone-system, which includes more than 300 zones in Europe and the rest of the world, is instead not described in details.

<sup>33</sup> There is an intermediate L<sub>1</sub> zone system which is applied in certain parts of the freight demand model.

less than 5.0 million combinations in total, which correspond to slightly more than one person per cell entry in this gross spatial-socio matrix. The elements of the different dimensions are shown in the Table 7-2 below.

**Table 7-2. The elements of the dimensions applied in the master table for individuals.**

<b>Dimension</b>	<b>Element</b>
Children	0
	1 or more
Age group	0-7
	8-14
	15-18
	19-24
	25-29
	30-54
	55-64
	65-74
	75-84
	85-
Gender	Male
	Female
Single	No partner
	Partner
Labour market association	Full-time employed
	Half-time employed
	Pupil or student
	Retired
	Unemployed, job seeking
	On social security, not job seeking
Income categories (1000 DKK)	0
	0-100
	100-200
	200-300
	300-400
	400-500
	500-600
	600-700
	700-800
	800-1000
	1000-

To forecast the master table in a given future year we define three main target tables as shown in Table 7-3 below. These tables are defined exogenously to the model system and are typically based on official forecasts, e.g. for the population growth, employment and general economic forecasts.

**Table 7-3. Targets applied in the population generator for individuals.**

Target constraint ID	Variable combination	Notation	Dimensions
TP <sub>1</sub>	Age×Gender×L0	TP <sub>1</sub> (a,g,L <sub>0</sub> )	1,960
TP <sub>2</sub>	Age×Income×L0	TP <sub>2</sub> (a,i,L <sub>0</sub> )	10,780
TP <sub>3</sub>	Age×Lmax×L0	TP <sub>3</sub> (a,l,L <sub>0</sub> )	5,880

The solution for the population fitting can be formulated as a matrix fitting problem. The solution space is defined by  $q=\{a,g,i,l,s,c,L_2\}$  with a dimensionality of 4,788,960. The solution to the population fitting problem can be expressed as the solution to the cross-entropy maximisation problem as shown in (41) below

$$\begin{aligned}
 \underset{\{t_q\}}{\text{Max}} Z_{TP} &= -\sum_q t_q \ln \left( \frac{t_q}{t_q^{\text{init}}} \right) - t_q \\
 \text{s.t.} \\
 \sum_{i,l,s,c} t_q &= TP_1(a, g, L_0), \forall a, g, L_2 \in L_0 \\
 \sum_{i,l,s,c} t_q &= TP_2(a, i, L_0), \forall a, i, L_2 \in L_0 \\
 \sum_{i,l,s,c} t_q &= TP_3(a, l, L_0), \forall a, l, L_2 \in L_0 \\
 t_q &\geq 0 \forall q
 \end{aligned} \tag{34}$$

The objective function is the cross-entropy where  $t_q$  is fitted with the initial matrix  $t_q^{\text{init}}$ . The targets TP<sub>1</sub>(a,g,L<sub>0</sub>), TP<sub>2</sub>(a,i,L<sub>0</sub>) and TP<sub>3</sub>(a,l,L<sub>0</sub>) represent the targets for a given forecast year<sup>34</sup>. The solution of the cross-entropy problem is intractable due to its large dimensionality when solved as a non-linear mathematical problem. Instead we use a dedicated Iterative Proportional Fitting (IPF) routine to solve the problem. This will yield to maximum likelihood estimates (Litle and Wu, 1991, Bishop et al., 1975) but be significantly faster with a runtime under 1 minute for this problem.

<sup>34</sup> There are interactions between the constraints which may lead to inconsistencies when these are updated. If for instance people are making adjustments to the targets separately it may be that summing over similar dimensions for different targets may not yield the same summation. To cope with this potential problem the model is designed with an internal harmonisation process, based on a ranking principle which makes sure that all constraints are consistent (Rich and Mulalic, 2012). This has no effect in the current experiment as we make sure that all tables are properly harmonised by construction.

### The household simulation stage

The outcome of the population fitting is a new master table which conforms to the new targets and to the structure of the master table for individuals. The next stage in the synthesis of the population is to allocate individuals into households. It is worth noting that the master table is not a micro representation of the population as it is a weighted list of prototypical individuals grouped into socio-groups. Although the socio-groups are relatively detailed, certain entries in the master table may represent as much as 30-50 prototypical individuals. If we apply micro-simulation directly on the master table it would mean that all of these 30-50 individuals would be treated in a similar way as regard the household sampling, which is not desirable. Due to this we create an enumerated list of all individuals in the population which are then processed in a micro-simulation loop in which individuals are grouped into households. The micro-simulation scheme is based on the following overall steps:

- 1) Extend the master table with a variable representing the adult status of the individual. This is based on a deterministic probability  $P_a$  of being adult based on the full set of socio-economic variables (income, age, labour market association and more). This table is referred to as an “extended master table” (EMT).
- 2) Construction of an aggregate household table (AHT) by summing the EMT according to <Zone ID, Single, Kids>. The counts then representing the sum for each household class rounded to nearest integer.
- 3) Let  $k=1,...,K$  represent the different aggregated household classes and  $N_k$  the number of households within each class. Initialise  $k=1$ .
- 4) Let  $i=1,...,N_k$  represent the individual households within each class. Initialise  $i=1$ .
- 5) For {i.k.} do the following;
  - a. Sample first adult.
  - b. If SingleID = 0 sample based on the first adult a second adult.
  - c. If SingleID = 1 go to 5d.
  - d. If KidsID=1 sample based on the household characteristics and the characteristics of the adults (there may be one or two) the expected number of kids  $NK_{ik}$  represented as an integer value.  
If  $NK_{ik} > 0$  sample  $1,...,NK_{ik}$  kids.
- 6) While  $i < N_k$  let  $i=i+1$  and go to 5). If  $i=N_k$  go to 7).

7) While  $k < K$  let  $k = k + 1$  and go to 4). If  $k = K$  go to 8).

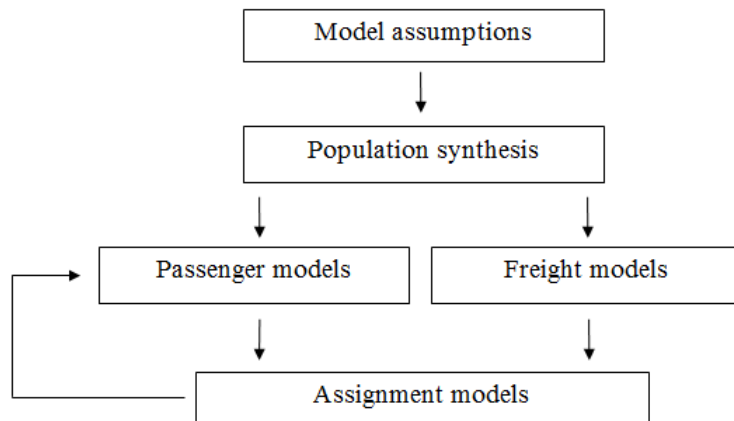
8) End of sampling.

A detailed description of the micro-simulation scheme is out of the scope of the present paper, however it should be said that step 5b) involves a spouse-match model where the marginal probabilities of selecting a spouse of a given type (classified according to income, age, labour market association and gender) depends on the characteristic of the “searching” individual. The model works similarly for the selection of the type of kids which depends on household income, characteristics of the adults as well as characteristics of the different kids.

The final list of individuals after joining these into households may not be entirely consistent with the master table when aggregating on various dimensions. This is because it is based on random draws, which although consistent at the aggregate level due to the law on large numbers may not be entirely consistent with the targets. To enforce consistency the list of individuals that result from the micro-simulation is re-scaled so that it matches the totals from the targets. It means that for a household one person may weight with say 1.1 and another one by 0.9 when used in the demand model, however, the weights are typically fairly close to one and

### **The NTM Transport demand model**

The NTM is a comprehensive transport demand model framework which has been developed for the Danish Ministry of Transport as a tool to be used for all transport projects evaluation in Denmark (Rich et al. 2010). The NTM model framework is illustrated in Figure 7-1 below.



**Figure 7-1. The Danish National Transport Model framework.**

At an initial step, exogenous assumptions are feed to the model. These assumptions include population forecasts represented as a list of people as described above, transport networks, public transport schedules, and employment to mention the most important inputs. Transport demand is then modelled in two parallel sections: (i) a passenger model, and (ii) a freight demand model. The output of these two models then feeds the assignment models which find the transport (stochastic) user equilibrium based on the calculated demand matrices. The output of the assignment models is the level-of-service represented as travel time matrices. The travel time is then feed back to the demand models in an iterative loop to reach equilibrium in the outer-loop between the demand and the assignment model. Currently this is accomplished through a heuristic approach based on a weighted method of successive averages.

The underlying transport models represent more than 18 different sub models for different trip segments, different trip durations and whereas the models involve trips outside Denmark. The model is tour-based and involves choices related to car ownership (at the household level), trip frequency (at the individual level), destination choice and choice of mode. A more elaborate description is beyond the scope of the current paper and as the final documentation of the NTM is not yet finished we refer to Rich et al. (2010).

### 7.3 Scenario analysis

The three scenarios we describe in the following paragraphs are based on official forecasts from SD and the Danish Ministry of Finance. The population target (TP<sub>1</sub> in Table 7-3) is taken directly from SD which every year publishes a projection broken down on municipalities to year 2040



and a national projection to year 2050. The future moving pattern (migration) between municipalities is based on the pattern from the latest four observed years. The projection of the income target (TP<sub>2</sub> in Table 7-3) is based on an observed 2010 target which is then projected with respect to a GDP growth. The first thing to do is therefore to consider GDP and how GDP are projected at the national level and then, based on this, calculated at municipal level<sup>35</sup>. The projection of employment and labour market association are based on an aggregate (i.e. 12 production sectors) national economic projection from the Ministry of Finance (the ADAM model) and the distribution of the employment of these 12 sectors on the 98 municipalities in the base year (2010). The projection is from April 2013 and refers to the “Konvergensprogram” from the Danish Ministry of Finance. An overview of the three different scenario definitions is given in Table 7-4 below.

**Table 7-4. The different population scenarios.**

Scenario	Description
Naïve	Forecast of the population where the socio-economic profile and the relative location do not change from 2010 baseline.
Socio	Forecast where the socio-economic dimensions are fitted and conform to future socio-economic margins. However, the relative location pattern at the level of municipalities is unchanged.
Spatial-Social	Forecast where the Spatial and Socio-economic dimensions are fitted. The forecasted then move in the geographical and socio-economic space to conform to future targets.

The “Naïve” scenario does not require the population fitting step. First we generate the 2010 population, and then simply re-scale all entries in the 2010 solution by the ratio k:

$$k = \frac{\sum_{a,g,L_0} TP_{1,2030}(a,g,L_0)}{\sum_{a,g,L_0} TP_{1,2010}(a,g,L_0)} \quad (35)$$

The “Socio” scenario allows the population and people to move within socio-economic classes, although, they are spatially distributed as in the 2010 population. In order to establish this scenario, we need to calculate new targets, which will be based on the marginal probabilities in 2030 across the socio-economic space. These are:

<sup>35</sup> More precisely, the GDP projection at the municipal level is not exactly a projection of GDP but a projection of Gross Value Added (GVA) using the municipal employment and national productivity by sector as a key. However, key figures are scaled so that they add up to the national GDP projection from ADAM (SD, 2013).

$$P_1(a, g | Z_0) = \frac{TP_{1,2030}(a, g, L_0)}{\sum_{a,g} TP_{1,2030}(a, g, L_0)} \forall L_0 \quad (36)$$

$$P_2(i, g | Z_0) = \frac{TP_{2,2030}(a, i, L_0)}{\sum_{a,i} TP_{2,2030}(a, i, L_0)} \forall L_0 \quad (37)$$

$$P_3(a, l | Z_0) = \frac{TP_{3,2030}(a, l, L_0)}{\sum_{a,l} TP_{3,2030}(a, l, L_0)} \forall L_0 \quad (38)$$

Targets are then defined as:

$$TP_{1,2030}^*(a, g, L_0) = k P_1(a, g | Z_0) \sum_{L_0} TP_{1,2010}(a, g, L_0) \quad (39)$$

$$TP_{2,2030}^*(a, i, L_0) = k P_2(a, i | Z_0) \sum_{L_0} TP_{2,2010}(a, i, L_0) \quad (40)$$

$$TP_{3,2030}^*(a, l, L_0) = k P_3(a, l | Z_0) \sum_{L_0} TP_{3,2010}(a, l, L_0) \quad (41)$$

Finally, the “Spatial-Social” scenario is where people are allowed to change not only the residential location in space but also the socio-economic groups. So, this is the baseline projection mode in the model.

### Illustration of population scenarios

As can be seen in Table 7-5, the Spatial-Social scenario which is based on the official forecast, results in a significant population increase of 23.3% in the largest cities between 2010 and 2030. For the Naïve and Socio scenarios the population increase in these cities corresponds to the general population growth of 7%. This results in 215,000 new citizens in these cities.

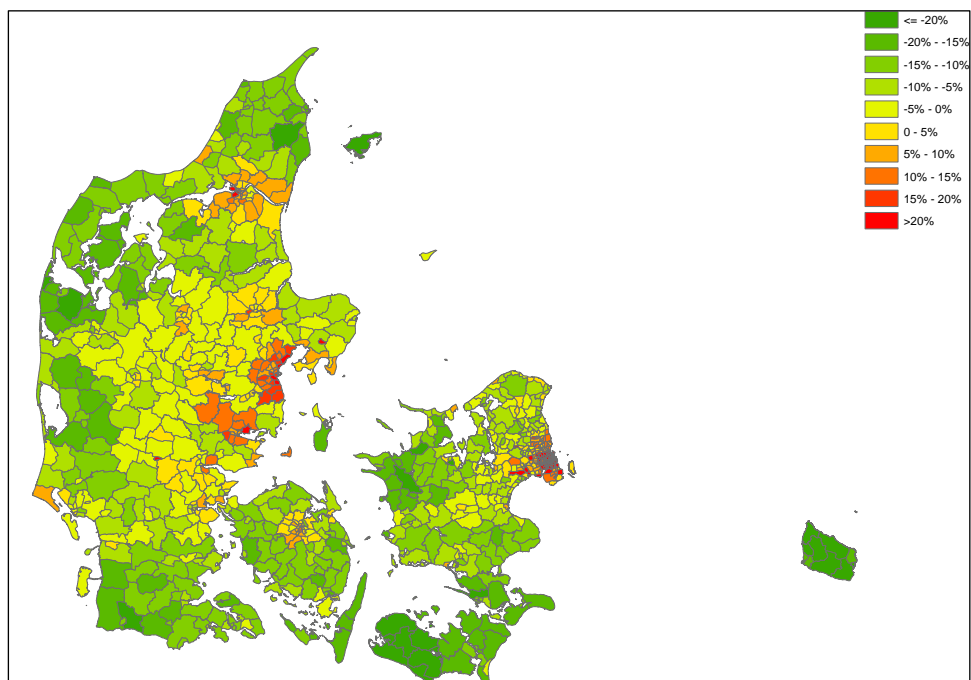
**Table 7-5. Population scenarios profiles.**

	Baseline 2010	Spatial-Social 2030	Naïve	2030	Socio	2030
Denmark	5,534,637	5,923,343	5,923,343		5,923,343	
Major cities*	1,336,165	1,647,450	1,430,017		1,430,017	

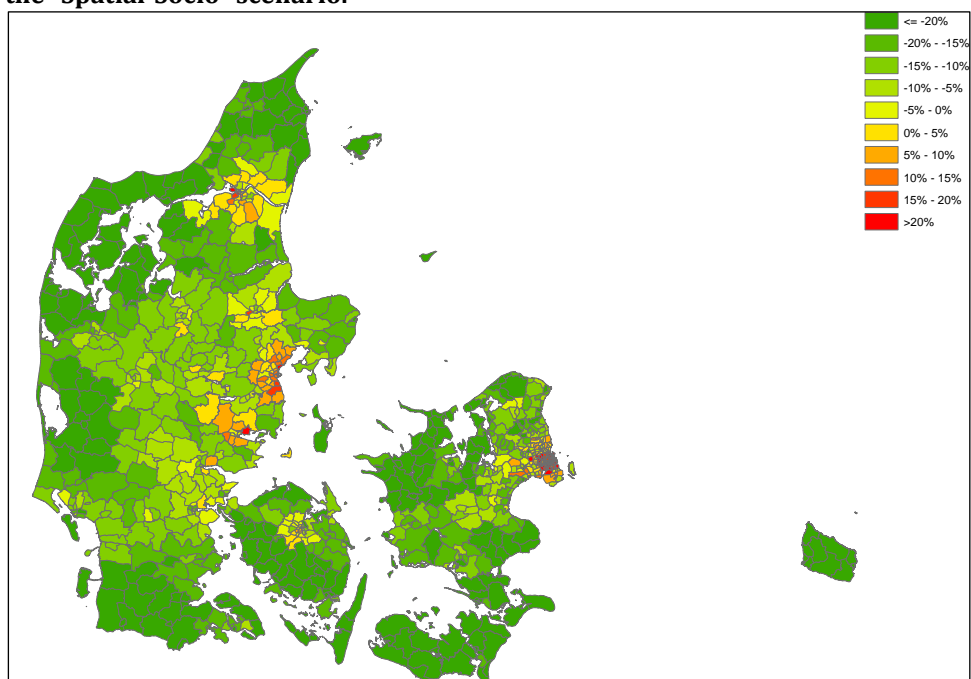
\*Copenhagen, Aarhus, Odense, Aalborg and Esbjerg.

The results in terms of changes of population density due to re-location are graphically shown in Figures 7-2 and 7-3. As can be seen by comparing the two figures, the “Spatial-Social”

scenario leads to higher population density in the main Danish urban areas, especially those of Aarhus and Copenhagen. This result is more evident when referring to the population in the age interval 19-64, as visually highlighted by the high number of red urban zones. In other words, while considering the overall population the changes of location toward the main urban areas may be compensated by new born or elderly moving back to the zones of origin in the countryside. Instead, when focusing the analysis on adults and active part of the population, this compensation does not produce effects.



**Figure 7-2. Percentage growth in population density in 2030 from the “Naïve” to the “Spatial-Socio” scenario.**



**Figure 7-3. Percentage growth in population density in 2030 from the “Naïve” to the “Spatial-Socio” scenario (age interval 19-64).**

## 7.4 Results

The effects of the changes of the population location and socio-economic composition on the transport demand are analysed by looking at the aggregate level and percentage changes. Results are shown in Tables 7-6 and 7-7. We look at the growth in number of trips, mileage travelled and average trip length with respect to the 2010 baseline.

**Table 7-6. Effects of changes in the population location and socio-economic composition on the transport demand (total).**

		Baseline	Spatial-Social	Naïve	Socio
Mode	Variable				
Walk	Trips	2,077,650	2,283,442	2,180,714	2,161,170
Bike		2,289,535	2,401,325	2,391,994	2,328,108
Car		6,308,321	6,976,481	6,963,513	7,132,122
Car passenger		2,905,071	2,745,312	2,867,474	2,798,284
Public		1,502,599	1,709,706	1,616,508	1,585,377
Air		996,174	985,702	996,003	996,498
Total		16,079,349	17,101,968	17,016,205	17,001,559
Walk	Mileage	4,264,449	4,324,404	4,440,415	4,385,847
Bike		7,642,571	7,619,222	7,906,374	7,617,217
Car		105,706,623	120,342,712	121,524,720	124,738,949
Car passenger		53,030,545	49,486,756	52,518,797	51,291,763
Public		29,639,580	32,834,988	33,446,165	32,650,221
Air		278,115,015	274,825,204	277,997,490	278,098,220
Total		478,398,784	489,433,286	497,833,961	498,782,217
Walk	Trip length	2.05	1.89	2.04	2.03
Bike		3.34	3.17	3.31	3.27
Car		16.76	17.25	17.45	17.49
Car passenger		18.25	18.03	18.32	18.33
Public		19.73	19.21	20.69	20.59
Air		279.18	278.81	279.11	279.08
Total		339	338	341	341

**Table 7-7. Effects of changes of the population location and socio-economic composition on the transport demand (percentage).**

		<b>Spatial-Social</b>	<b>Naïve</b>	<b>Socio</b>
<b>Mode</b>	<b>Variable</b>			
Walk	Trips	9.91%	4.96%	4.02%
Bike		4.88%	4.48%	1.68%
Car		10.59%	10.39%	13.06%
Car passenger		-5.50%	-1.29%	-3.68%
Public		13.78%	7.58%	5.51%
Air		-1.05%	-0.02%	0.03%
Total		6.36%	5.83%	5.74%
Walk	Mileage	1.41%	4.13%	2.85%
Bike		-0.31%	3.45%	-0.33%
Car		13.85%	14.96%	18.00%
Car passenger		-6.68%	-0.97%	-3.28%
Public		10.78%	12.84%	10.16%
Air		-1.18%	-0.04%	-0.01%
Total		2.31%	4.06%	4.26%
Walk	Trip length	-7.73%	-0.79%	-1.13%
Bike		-4.95%	-0.98%	-1.98%
Car		2.94%	4.15%	4.37%
Car passenger		-1.25%	0.33%	0.41%
Public		-2.64%	4.89%	4.41%
Air		-0.13%	-0.03%	-0.04%
Total		-0.28%	0.47%	0.44%

From Tables 7-6 and 7-7 several things are revealed, as summarised below:

- The Spatial-Social has the higher increase of total number of trips, of 6.36%, while the increase for the Naïve and Socio scenarios are, respectively, 5.83% and 5.74%. At the same time the increase of total mileage for the Spatial-socio is of only 2.31% whilst for the other two scenarios is 4.06% and 4.26%. The reason for such results is that the average trip length decreases in the Spatial-socio of -0.28%, while increases in the other two scenarios of 0.47% and 0.44%.
- While the number of trips made by car increases at similar rate in all three scenarios, the number of trips for walk and public transportation shows a peak in the Spatial-Social, of 9.91% and 13.78%, the double than shown from the other two scenarios.

- On overall, the mileage changes follow the variations in the number of trips by mode. The more evident changes are related to the car and public transport modes, which respectively increase 15.60% and 11.26%, as average of the three scenarios.
- In the Spatial-Social scenario the mileage variation for trips made on foot of 1.41% and bike -0.31% does not match the increase in corresponding number of trips of respectively 9.91% and 4.88%; this is due to the shorter distances covered, corresponding to a decrease of -7.73% and -4.95% in trip length.
- When comparing the Naïve and the Spatial-Social, the average trip distances are consistently over-estimated for all modes in the Naïve scenario. Most significantly for public transport which have 7.7% longer distances in the native experiment. Instead, there are not large differences between Socio and the Naïve scenario indicating that the change in social profile does not largely affect trip distances.

## 7.5 Summary and conclusion

This paper investigates the effects of urbanization and social mobility on the demand of transport. In particular, the analysis focused on the hypothesis, based on the observed worldwide trend, that the employed, richest, and younger to middle-aged part of the population migrates to urban areas. Using Danish demographic data, three scenarios were created simulating alternative social and spatial conditions for the year 2030: i) Naïve, simulating only population growth, (ii) Socio, simulating population growth and social mobility, and iii) Spatial-Social, simulating population growth, social mobility and location mobility, so representing the on-going urbanisation process. Sensitivity tests have been implemented using the new Danish national transport model. The effects have been analysed in terms of variation of total number of trips, total mileage travelled and average trip distance, as compared to the 2010 reference case.

The results show that the combined effect of higher urban density and social mobility, represented in the Spatial-Social scenario, produces an increase of number of trips. This is due both to the higher number of opportunities, e.g. recreational, social, educational, etc., which a dense and mixed urban form offers, and to the more active behaviour associated to younger and richer individuals and households. Among the two factors shaping the scenario, density and social mobility, the first seems to be dominant. In fact, the differences in terms of generated trips between the other two scenarios, Naïve and Socio, are instead not noticeable at aggregated

level. Furthermore, in the Spatial-Social scenario the combined effect of higher levels of urban density and the assumptions made in terms of social mobility lead to an increase of both trips made on walk, due to shorter distances, and by public transport, due to re-bound effects from higher cost of travelling by car. This is consistent with what expected based on assumptions and previous studies on the topic.

Despite showing the higher increase in number of trips, the Spatial-Social scenario produces the lower increase in mileage travelled, whilst the higher is related to the Naïve scenario. This is due to the differences in average trip length. In the Spatial-Social the proximity of the destinations reduces, so decreasing the average trip length and consequently the total mileage travelled. This can be considered a direct consequence of the higher level of urban density in the Spatial- Social, which implies closer residence, work and leisure activities locations. On the contrary, in the Naïve scenario, which represents the larger degree of sprawl (socially and spatially), the trips are longer compared with the Spatial-Social.



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## 8 Conclusions

The present thesis described the results from four studies on uncertainty in transport models and forecasts. Transport planning and projects evaluation are based on travel demand forecasts, output of transport models. However, transport models reproduce complex systems hence they have inherent uncertainty which prevents the modeller from using a deterministic approach when reproducing those systems in the modelling process. The consequence of such uncertainty is that model point output becomes unreliable, being only one of the possible model outputs. Instead, to be informative for the decision making process, model output should be represented as a probability distribution, with a range of possible values and corresponding likelihood of occurrence. Uncertainty can be found in any of the transport model components, such as the socioeconomic input, calibrated parameters and model assumptions (context) and structure to finally reflect in the model outputs. Despite this to be a known fact, in the transport modelling literature uncertainty is not often investigated and the aim of this study is to fill some of the existing gaps.

The uncertainty analyses described in the present thesis were carried out by implementing scenario analysis and stochastic simulation. The reason to choose these methods was twofold. First, it was decided to choose methodologies known or straightforward to interpret for all the stakeholders potentially involved in a hypothetical decision process. Both sensitivity test and stochastic simulation, especially MCS, have these characteristics, given that they are commonly used in different fields of knowledge. Second, unlike other uncertainty analysis methods, such as analytic expressions, scenario analysis and stochastic simulation can be implemented on any model, irrespective from its structure, calibration method, etc. This guarantees high flexibility of these approaches and the possibility of replicating the analyses illustrated in this thesis on any model.

Two were the models used as case studies: a “classical” four-stage framework, the Næstved model, and an activity based model, the NTM. The Næstved model was chosen because, despite some criticisms, the four-stage model is still widely used, so the results from that study can be used for comparative purposes. The NTM was instead chosen because for some of the analyses, such as the ones on uncertainty in forecasts and spatial distribution, the use of a state of art national model clearly helped in obtaining realistic and meaningful results. It is however important to stress that although the results from any transport model uncertainty analyses

provide insights and can be used for comparative purposes, they cannot be fully generalised. In fact, transport models differ by context, structure, inputs, etc. thus specific uncertainty analysis should be implemented for different models so to address the specific uncertainties of each model.

The results of the present research are various. The work on the BPR formula parameters, showed how their variability is an important source of uncertainty for transport models, if not for an entire network, definitely for selected links. In particular, in the case study the links affected mainly refer to road types potentially hosting commuting traffic. Any assessment of projects potentially affecting the traffic flows in the areas where those links are located should then integrate speed-flow curves uncertainty analysis in the modelling framework. Similarly, the results that showed that the higher is the link congestion, or the link traffic volumes (holding the assumption of independency), the lower is the level of uncertainty, offer some guidance in terms of the (higher) risk inherent traffic forecasts referring to non-congested areas of the network.

Also the increase of model outputs uncertainty over time, especially with respect to congested travel time, should be a reason of concern for projects and policies requiring long time period to reach the break-even point between costs and benefits. The combined effect of higher urban density and social mobility produces an increase of number of trips. Among the two factors shaping the scenario, density and social mobility, the first seems to be dominant. However, despite increasing the number of trips, urban density increases the proximity of the destinations so decreasing the average trip length and the total mileage travelled. The assumptions on the future spatial distribution and social characteristics of the population living in the modelled area then result of key importance and a big source of uncertainty for the modelling process.

From a methodological perspective, both the analyses on the choice of the probability distribution functions to use to implement a stochastic sampling procedure and on the uncertainty propagation over time provided potentially valuable results. With respect to the first study, the correspondence that has been found between variables distribution and model output distribution suggests that a lower number of model runs might be required to obtain informative output, so resulting in relevant time savings. The second study was instead based on the idea that the further in the future the model forecasts, the higher should be the

uncertainty inherent the model itself. With respect to the forecasts of the model inputs, two methods have been suggested to represent this growing uncertainty based on the idea of including a higher range of possible events (represented by the variance derived from the variable time series) the longer the model forecasts.

Uncertainty analysis has a number of weaknesses that have to be clearly stated. Uncertainty analysis does not improve the quality of a model, although it helps in identifying some of the model weaknesses. Neither, similarly, uncertainty analysis improves the accuracy of a model, given that the most likely model output, to be considered as reference case for the assessment framework, is still the model point output. Furthermore, the results from model uncertainty analysis cannot be validated. In fact, in order to validate the probability distribution of the model output, resulting from the implementation of an uncertainty analysis, the “observed” probability distribution of the system output should be available. This is of course impossible, given that “observed” probability distribution of the system output would include all the events that have never occurred in reality. However, the results from this thesis – consistently with those from existing literature on the topic - show that uncertainty highly affects transport models output. The decision making processes that do not take into account this uncertainty increase the risk inherent to the decision they generate. Instead, uncertainty analysis allows identifying and quantifying the main sources of uncertainty within the model so providing knowledge on the level of confidence of the model output. This enhances not only the robustness of the travel demand models but also of the decisions taken based on their outputs. In fact, only when supported by the results from model uncertainty analysis, transport models output can provide enough information to allow a thorough transport projects and policy assessment and the development of adaptive or robust plans.